Inferences and Observations of Turbulent Dissipation and Mixing in the Upper Ocean at the Hawaiian Ridge

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ABSTRACT

The Hawaiian Ridge is one of the most energetic generators of internal tides in the pelagic ocean. The density and current structure of the upper ocean at the Hawaiian Ridge were observed using SeaSoar and Doppler sonar during a survey extending from Oahu to Brooks Banks and up to 200 km from the ridge peak. Survey observations are used to quantify spatial changes in internal-wave-induced turbulent dissipation and mixing. The turbulent dissipation rate of kinetic energy \( \epsilon \) and diapycnal eddy diffusivity \( K_p \) are inferred from an established parameterization using internal wave shear as input. At the Kauai Channel (KC) and French Frigate Shoals/Brooks Banks sites, \( \epsilon \) and \( K_p \) decay away from the ridge with maxima exceeding minima by 5 times. At both sites, average \( K_p \) is everywhere greater than the canonical open-ocean value of \( 10^{-5} \text{m}^2\text{s}^{-1} \). Along the ridge, \( \epsilon \) and \( K_p \) vary by up to 100 times and are largest at sites of largest numerical model internal tide energy density. In the eastern KC, \( K_p > 10^{-3} \text{m}^2\text{s}^{-1} \) is typical in a patch more than 200 m thick located above the path of an \( M_2 \) internal tide ray. An upper limit on the dissipation rate from \( M_2 \) internal tides to turbulence within 50 km of the Hawaiian Ridge is roughly estimated to be in the range of 4–9 GW. At KC, the depth-integrated internal wave energy density and dissipation rate are positively correlated. Potential density inversions occur near the main ridge axis at significant topographic features. Average \( K_p \) is larger inside inversions.

1. Introduction

Turbulent diapycnal mixing is a necessary component of the meridional overturning circulation of heat and salt (Munk and Wunsch 1998; Wunsch and Ferrari 2004). In midlatitudes far from convection sites, and at middelshes beneath the wind mixed layer and subduction zones and above the bottom boundary layer, diapycnal mixing is the dominant mechanism for transporting heat vertically through the water column. The diapycnal eddy diffusivity \( K_p \) is geographically nonuniform in the pelagic ocean. Observations of a tracer introduced at 300-m depth in the open-ocean midlatitude Atlantic give a direct estimate of \( K_p \) of \( 1.1 \pm 0.2 \times 10^{-5} \text{m}^2\text{s}^{-1} \) (Ledwell et al. 1993). Midlatitude open-ocean velocity microstructure measurements indicate that \( K_p \) is no larger than \( 5 \times 10^{-5} \text{m}^2\text{s}^{-1} \) in the upper 1000 m (Gregg et al. 2003). In contrast, at significant topographic features such as ridges and seamounts, upper-ocean \( K_p \) is 10 or more times as large (Lueck and Mudge 1997; Kunze and Toole 1997). Inverse models of the mean midlatitude vertical density structure (Munk and Wunsch 1998) and property budgets for bounded regions (see Table 1 in Wunsch and Ferrari 2004) require \( K_p \approx 10^{-4} \text{m}^2\text{s}^{-1} \). The outstanding and oft-posed question then is whether enhanced mixing at small areas of significant topography is enough to eliminate the discrepancy between predicted and observed \( K_p \) in the open ocean. The appropriateness of this question has been challenged because estimates of \( K_p \) made from observations taken over a month or less leave out processes such as mesoscale eddies, which could contribute to the predicted \( K_p \) (Davis 1994). Regardless, diapycnal mixing is larger at significant topographic features than in the open midlatitude ocean. An improved understanding of how mixing decays between a significant topographic feature and the open ocean has implications for how mixing is included in global models.

One significant topographic feature where enhanced mixing occurs is the Hawaiian Ridge. Local dissipation of internal tides generated at the ridge is hypothesized...
to be the primary energy source for enhanced mixing. A tidal model that assimilates satellite altimeter measurements indicates the energy loss rate of the barotropic lunar-semidiurnal (\(M_2\)) tide to internal tides or bottom friction is \(19^{+6}_{-1.5}\) GW (Zaron and Egbert 2006). The uncertainty results from testing the model’s sensitivity to a forcing parameter. Approximately 17 GW is lost within 250 km of the ridge. Numerical model results indicate approximately 10-GW fluxes away from the ridge at the 4000-m isobath as \(M_2\) internal tides (Merrifield and Holloway 2002, hereinafter MH02). Comparison with observations indicates this across-4000-m-isobath flux could be 10 \(\pm\) 5 GW (Rudnick et al. 2003). MH02 find the 4000-m isobath encloses the major internal tide generation regions and that the direct loss of barotropic tides to bottom friction is negligible. Thus, most of the 17 GW may be converted to internal tides within the 4000-m isobath. The rate of \(M_2\) internal tide dissipation within the 4000-m isobath is the difference between the barotropic loss rate and the outgoing internal tide flux and thus is predicted to be \(7^{+10.5}_{-6.5}\) GW.

Using a variety of microstructure observations from the Hawaii Ocean Mixing Experiment (HOME) to create an empirical function for the three-dimensional spatial structure of \(K_w\), Klymak et al. (2006, hereinafter K06) estimate 3 \(\pm\) 1.5 GW of tidal energy is lost to turbulence within 60 km of the ridge (near the 4000-m isobath at the most important dissipation sites).

As part of HOME, we conducted a survey of the Hawaiian Ridge obtaining SeaSoar conductivity–temperature–depth (CTD) and ship-based Doppler sonar velocity observations (Fig. 1). We use the observations to quantify and study the spatial structure of internal wave dissipation and mixing in the upper ocean. The survey goals were to observe the transition from enhanced mixing near the ridge to open-ocean levels and to learn about variations in mixing along the ridge. Details of the observations are discussed in section 2.

The internal wave energy available for mixing varies greatly around the Hawaiian Ridge. In the upper ocean, internal wave energy density changes by a factor of 10 over across-ridge distances of 100 km (Martin et al. 2006).
2. Data

The SeaSoar/Doppler sonar survey was made between 29 August and 29 September 2000 (Fig. 1). Two along-ridge surveys followed approximately the 1000-m isobath between the island of Oahu and BB. Across-ridge surveys were made at FFS/BB and at KC. Motivated by the observation of large isopycnal displacements, one leg of the survey at KC was repeated 8 times. An across-ridge survey west of the island of Kauai and a combined along/across-ridge survey were also made. The survey-sampling pattern did not resolve semidiurnal or other frequencies associated with internal waves. In the following sections the survey observations are used to investigate the average contribution of internal waves to mixing. SeaSoar was flown in a vertical sawtooth pattern between about 20- and 370-m depth at a tow speed of 4.1 m s⁻¹. The SeaSoar/Doppler sonar data processing is discussed in MRP06. Density inversions are investigated using the SeaSoar CTD data averaged to 1-s intervals. Estimates of $\epsilon$ from the parameterization are made using the CTD and 50-kHz Doppler sonar data grid averaged to 15-min time intervals and 6.15-m depth intervals.

3. Parameterization estimate of the dissipation rate and diapycnal eddy diffusivity

The turbulent dissipation rate of kinetic energy is estimated using the procedure of Gregg (1989). This estimate, $\epsilon_{IW1}$, is obtained at each point in the SeaSoar/Doppler sonar grid and is proportional to the fourth power of shear computed from first differences. The diapycnal eddy diffusivity is computed using $K_p = 0.2\epsilon_{IW1}/N^2$, where $N$ is the buoyancy frequency, the result of a balance in turbulent kinetic energy among turbulent production, work against buoyancy, and dissipation (Osborn 1980). The procedure for computing $\epsilon_{IW1}$ and the justification for using $\epsilon_{IW1}$ are in the appendix. The cruxes of the appendix are 1) there are three established procedures for estimating $\epsilon$ from finescale shear (and from strain in one procedure); 2) of these, the $\epsilon_{IW1}$ procedure produces best agreement with microstructure shear measurements made during HOME; 3) on average, $K_p(\epsilon_{IW1})$ agrees with $K_p$ derived from microstructure profiler data to within a factor of 2. Hereinafter, $\epsilon_{IW1}$ is simply referred to as $\epsilon$. Shear measurements from the same Doppler sonar have been used previously to infer dissipation through the Gregg
parameterization in the Kuroshio Current (Rainville and Pinkel 2004).

The diffusivity $K_p$ is frequently larger than $10^{-5} \text{ m}^2\text{s}^{-1}$. Particularly large values are observed during a crossing of the Kauai Channel at the peak of spring tide [$M_2$ and $S_2$ (solar-semidiurnal) barotropic tidal constituents in phase] (Fig. 2a). These large values are potentially the result of internal tide dissipation. MRP06 observed a region of enhanced internal wave kinetic energy density along an across-ridge path extending from the northern edge of the ridge peak in the eastern Kauai Channel to a point at the surface on the south side of the ridge. This path is consistent with an $M_2$ internal tide ray along which internal tide energy propagates. Additional evidence for a ray comes from the observation of enhanced southward semidiurnal internal wave energy flux along the same path (Nash et al. 2006; Rainville and Pinkel 2006). In Fig. 2a, $K_p$ is largest above where the ray intersects the section. The $K_p$ is typically in excess of $10^{-3} \text{ m}^2\text{s}^{-1}$ over 200 m above the ray and $K_p$ of $10^{-2} \text{ m}^2\text{s}^{-1}$ is not uncommon. Much lower $K_p$ is observed during a second crossing at the end of the neap half of the spring-neap cycle (Fig. 2b).

Figure 2a demonstrates strong mixing, as predicted by the $K_p$ estimate, occurs near the Hawaiian Ridge. The strong mixing observed near the ray at the peak of spring tide suggests the mixing results from the dissipation of semidiurnal internal tides. In sections 3a and 3b,
we investigate the across- and along-ridge spatial structure of enhanced dissipation and mixing present near the Hawaiian Ridge. In section 3c an attempt is made to determine the total dissipation rate within some distance of the ridge by integrating $\epsilon$.

a. Across-ridge structure

In this section, data from the across-ridge surveys at the KC and FFS/BB sites are used to investigate the spatial structure of average $\epsilon$ and $K_p$. Two averages are used. The first average, denoted by $\langle \rangle$, is a horizontal mean over all profiles that fall within bins 10 km wide in position normal to the ridge. The second average, denoted by $\langle \rangle_d$ is a vertical mean over the depth range (well below the mixed layer base) to 550 m (the range of the Doppler sonar). Results of this averaging are shown in Fig. 3 for KC and Fig. 4 for FFS/BB. Mean-square shear magnitude $\langle S^2 \rangle_d$ is shown in Figs. 3a and 4a. The shear magnitude squared ($s^{-2}$) is computed from first differences:

$$S^2 = [(\Delta u/\Delta z)^2 + (\Delta v/\Delta z)^2].$$

The difference interval $\Delta z$ is the vertical grid spacing of 6.15 m and $u$ and $v$ are, respectively, the east and north components of velocity from the Doppler sonar.

Across-ridge trends in $\langle e \rangle_d$ and $\langle K_p \rangle_d$ are present on both sides of the ridge at KC and on the south side at FFS/BB (Figs. 3b,c and 4b,c). The trends show maximum values of $\langle e \rangle_d$ and $\langle K_p \rangle_d$ exceeding minimum values by 5 or more times. At all positions at both sites, $\langle K_p \rangle_d$ exceeds the canonical observed open-ocean value of $10^{-5}$ m$^2$ s$^{-1}$. At KC, $\langle e \rangle_d$ and $\langle K_p \rangle_d$ are maximum between positions $-25$ and 5 km. At FFS/BB, we did not repeatedly survey within 15 km of the ridge peak because of the shallow topography, and cannot determine how large average mixing is there. Far from the ridge on the north side, $\langle e \rangle_d$ and $\langle K_p \rangle_d$ are larger at FFS/BB. On the south side, $\langle e \rangle_d$ and $\langle K_p \rangle_d$ have larger maxima at FFS/BB than at KC. Large $\langle K_p \rangle_d$ is often observed below 300 m, and $\langle K_p \rangle_d$ outside of maxima is typically larger at FFS/BB (Figs. 3d and 4d). Please note these averages are biased relative to the spring-neap cycle. Across-ridge surveys at KC occurred during spring tide and across-ridge surveys at FFS/BB during neap tide.

The across-ridge gradient of $K_p$ is an important quantity for understanding the length scale separating locations of enhanced mixing such as Hawaii from the open ocean. We estimate the across-ridge gradient $d[\log_{10}(\langle K_p \rangle_d)]/dx$ at KC and FFS/BB by using exponential fits to $\langle K_p \rangle_d$ (Figs. 3c and 4c). Choice of an exponential function is not based in theory, but made to obtain some measure of decay scale. At KC one exponential curve is fit from position $-170$ km to $x_o$, the position of maximum $\langle K_p \rangle_d$, and the second curve from $x_o$ to position $170$ km. On the south side of FFS/BB, an exponential curve is fit from position $-120$ km to the maximum. For completeness, an exponential is fit between 20 and 120 km on the north side of FFS/BB, even though $K_p$ does not decay there. South of the maximum at KC, $d[\log_{10}(\langle K_p \rangle_d)]/dx$ is 0.58 (100 km)$^{-1}$. This gradient corresponds to a factor of 3.8 change in $\langle K_p \rangle_d$ over 100 km and to an $e$-folding length scale of 76 km. North of the maximum at KC, $d[\log_{10}(\langle K_p \rangle_d)]/dx$ is $-0.30$ (100 km)$^{-1}$, corresponding to a factor of 2.0 change in $\langle K_p \rangle_d$ over 100 km and an $e$-folding scale of 147 km. South of the maximum at FFS/BB, $d[\log_{10}(\langle K_p \rangle_d)]/dx$ is 0.54 (100 km)$^{-1}$, corre-
operator is a mean over depths $M$ (filled squares). The dashed lines are exponential function fits from position 20 and 120 km between positions 20 and 120 internal tide generators, and 3) across ridge over 100 km and an $\epsilon$-folding scale of 80 km. North of the maximum at FFS/BB, $d[\log_{10}(\langle K_p \rangle_d)]/dx$ is 0.12 (100 km)$^{-1}$, corresponding to a factor of 1.3 change in $\langle K_p \rangle_d$ over 100 km and an $\epsilon$-folding scale of 372 km.

**b. Along-ridge structure**

Changes in $K_p$, with geographic location can be investigated using a map (Fig. 5). The quantity displayed in the map is $\log_{10}(\langle K_p \rangle_{d})$ where $\langle \rangle_{d}$ is a mean over all profiles that fall within regularly spaced 20 km $\times$ 20 km boxes and over the depth range of 100–550 m. The middle value of the color bar in Fig. 5 is near the mode of the probability density function (PDF) of $\log_{10}(\langle K_p \rangle_{d})$. More than 95% of the values shown in Fig. 5 fall within the color bar range. The geographic distribution of $K_p$ is nonuniform. The observed locations of largest mixing can be compared with the predicted locations of largest $M_2$ internal tide generation along the Hawaiian Ridge. Specifically, our comparison is with MH02’s map of $M_2$ internal tide energy density (their Fig. 3). The collocation of largest energy density from MH02 with largest $\log_{10}(\langle K_p \rangle_{d})$ is indicative of localized dissipation of $M_2$ internal tides. There is collocation at FFS/BB, west of Nihoa Island (centered at about 163°W), and in the eastern KC. Over the section of ridge observed in the survey, FFS/BB appears to be the site of overall largest generation and dissipation. There is also collocation between quieter regions such as 165°–164.5°W and 162.5°–161.5°W. There are differences, however. For example, there is enhanced mixing at the seamount at 161°W and south of Kauai, both places in MH02 without strong internal tide generation. In some boxes far from the ridge, large $\log_{10}(\langle K_p \rangle_{d})$ is obtained from a single ship pass. It is unknown how $\log_{10}(\langle K_p \rangle_{d})$ in these boxes would change with additional passes. Changing the size of the boxes to have sides 10 or 40 km does not alter conclusions about the locations of enhanced mixing. Because the survey required a month to complete, biases relative to the spring–neap cycle are necessarily included in the map.

**c. Integrated dissipation**

The turbulent dissipation rate integrated over the Hawaiian Ridge is an important quantity because it is an upper limit on the $M_2$ internal tide energy dissipation rate and thus on the $M_2$ internal-tide-induced mixing rate at the ridge. The integral is an upper limit on the $M_2$ internal tide dissipation rate because $M_2$ internal tides are likely not the only source of nonzero $\epsilon$ near the ridge. Non-$M_2$ internal tides, near-inertial internal waves, and longer time-scale (mesoscale) processes could also contribute to $\epsilon$. The relative contributions of $M_2$ internal tides, non-$M_2$ internal tides, and near-inertial internal waves to shear observed during the survey cannot be determined because of the lack of temporal resolution. At KC, values of the squared mean shear $[\langle (\Delta u/\Delta z)^2 + (\Delta v/\Delta z)^2 \rangle_{d}]$ are $<$7 $\times$ 10$^{-6}$ s$^{-2}$, smaller than the smallest mean-square shear values in Fig. 3a. Thus, mean shear, potentially the result of mesoscale processes, does not significantly contribute to the $\epsilon$ estimates. Ideally, the rate of energy transfer from $M_2$ internal tides generated at the ridge to turbulence would be determined by computing a full volume integral of $M_2$ internal-tide-driven $\epsilon$ multiplied by density ($\rho$). This volume integral would extend 1) over all depths, 2) along all sections of the ridge that are significant $M_2$ internal tide generators, and 3) across ridge from the ridge peak to the position beyond which internal tides persist as stable low modes. Here, we apply the mean across-ridge structure of depth-integrated $\rho \epsilon$.
to the entire ridge to obtain a total upper-ocean dissipation rate. The first step in the integration is to obtain the depth integral of $\rho e$ at each position in the entire SeaSoar/Doppler sonar survey. The depth integral over the vertical coordinate $z$ is defined by

$$\int_{-d_T}^{-d_B} \rho e \, dz.$$  

The integration limits are $d_T = 100$ m and $d_B = 550$ m. Significant dissipation of internal tides could occur shallower than 100 m, but the parameterization does not apply in the mixed layer. The depth integrals are then averaged into bins 10 km wide in position normal to the ridge (the $\{\}$ average). The cumulative integral over across-ridge position $x$ is computed by integrating from position 0 outward to each south side $x$ and to each north side $x$ (Fig. 6a). Bootstrapping is used to determine confidence bounds on $\left[ \int_{-d_T}^{-d_B} \rho e \, dz \right]$. The 95% confidence bounds for each across-ridge bin are obtained via the percentile method using 2000 bootstrap replications (Efron and Tibshirani 1993; Zoubir and Boashash 1998). Error bounds on the integrated dissipation are then established by using the lower and upper confidence bounds on $\left[ \int_{-d_T}^{-d_B} \rho e \, dz \right]$ in the Fig. 6a integrals. A simplistic estimate of the upper-ocean dissipation rate $D$ within a distance $d$ of the ridge is obtained by adding the values in Fig. 6a at positions $-d$ and $d$ and then multiplying by a ridge length of 2500 km. The 2500-km ridge length is the length over which most of the $M_2$ barotropic tidal loss and internal tide generation occurs (Egbert and Ray 2000; MH02). However, $M_2$ internal tide generation is predicted to be strongest at the locations sampled most heavily in the survey (KC, FFS/BB). The survey only covered a 1000-km section of ridge, but one in which mixing is expected to be larger than in the other 1500 km. The values in Fig. 6a are likely larger than those that would be obtained by including profiles evenly distributed over the entire 2500-km ridge. Thus, because of this bias and the unknown contribution from other sources, $D$ can be thought of as an upper limit on the total dissipation rate from $M_2$ internal tides to turbulence in the upper ocean within distance $d$ of the 2500-km ridge. For $d = 170$ km, $D = 11.4^{+5.5}_{-4.4}$ GW.

Fig. 5. A map of $\log_{10}(K_{1b})$ where $K_{1b}$ is a mean over all profiles that fall within regularly spaced 20 km $\times$ 20 km boxes and over the depth range 100–550 m. Observations from the entire cruise track shown in Fig. 1 are used.
The integrated turbulent dissipation rate can be compared to the $7^{+0.5}_{-6.5}$ GW of the $M_2$ internal tide energy predicted to be available for dissipation within the 4000-m isobath (see section 1). At numerous locations along the ridge, the 4000-m isobath is roughly 50 km from the ridge peak. Integration out to 50 km using the values in Fig. 6a gives $D = 4.1^{+1.3}_{-1.5}$ GW. Using K06’s structure function for $K_e$ as a function of across-ridge position and depth [Eqs. (A17)–(A19)] and $K_e = 0.2e/N^2$, the integrals in Fig. 6a are extended to the ocean floor by accounting for the dissipation occurring below 550 m (beyond the range of the Doppler sonar; Fig. 6b). For the extension, $N^2$ is from the KC profile in K06’s Fig. 1. The contribution below 550 m is $D_{\text{deep}} = 2.2 \pm 1.1$ GW. The uncertainty is the 50% error assigned by K06. The resulting full-depth dissipation rate within 50 km of the ridge is $D_{\text{fd}} = 6.3^{+2.7}_{-2.4}$ GW. Considering uncertainties in $D$ and $D_{\text{deep}}$ in the depth range 100–550 m contributes 46%–84% of the full-depth total. The structure function’s upper-ocean dissipation rate, obtained through the same procedure used to get $D$, is $D_{\text{struct}} = 3.2 \pm 1.6$ GW. The 3.2-GW value is less than $D = 4.1$ GW, because across-ridge gradients of upper-ocean $\epsilon$ are smaller in the survey data than in the structure function (Fig. A2). However, $D_{\text{struct}}$ and $D$ agree within the stated error bounds. The full-depth integral from the structure function alone is $D_{\text{fd}}^{\text{struct}} = 5.4 \pm 2.7$ GW, which is larger than K06’s stated value of $3 \pm 1.5$ GW.

This discrepancy is not unexpected, however, since K06 used a different integration method, allowing the integrand to be a function of along-ridge position.

Several sources contribute to uncertainty about how well $D_{\text{fd}}$ estimates the upper limit on the total dissipation rate from $M_2$ internal tides to turbulence within 50 km of the Hawaiian Ridge. First, statistics derived from the survey data have errors associated with the survey sampling. The survey data have nonuniform spatial distribution. Mixing is thought to be highly intermittent, and so the survey sampling may miss some large mixing events. There are also sampling biases relative to the spring-neap cycle as discussed above. The stated $^{\pm2}_{\pm4}$ GW uncertainty in $D_{\text{fd}}$ is one measure of statistical error as this uncertainty relies on bootstrapping and inclusion of the structure function’s statistical error. Second, comparisons with microstructure profiler data in the appendix (see Table A1) indicate $\epsilon_{\text{prof}}$ could be 0.6–1.5 times the true turbulent dissipation rate. Thus, the upper-ocean part of $D_{\text{fd}}$ could be 0.6–1.5 times the true integral. Third, although not expected, measurements from west of BB or east of Oahu could have increased the integrals in Fig. 6a and thus $D_{\text{fd}}$, resulting in a larger upper limit.

Estimating the fraction of $D_{\text{fd}}$ due to $M_2$ internal tides requires knowledge of the frequency distribution of the survey-observed shear. The survey data lack temporal resolution, so consideration of HOME observations with temporal resolution is necessary. Frequency spectra of 10-m shear, obtained from the Floating Instrument Platform (FLIP) when it was situated both over the KC ridge (near-field site) and more than 400 km south of Oahu (far-field site), have maxima between near-inertial and diurnal frequencies in the upper 800 m (Rainville 2004). A frequency spectrum of upper-ocean shear, obtained by operating the same Doppler sonar used in our survey over the KC ridge, has a maximum at diurnal frequency (Carter and Gregg 2006). The relative contributions of the near-inertial/diurnal band and the semidiurnal band to shear are estimated by integrating the frequency spectra of shear appearing in Figs. 2.7d–2.8d in Rainville (2004) and Fig. 4a in Carter and Gregg (2006). The near-inertial/diurnal band starts at the local inertial frequency and the semidiurnal band is centered around $M_2$. The bands have equal bandwidth, chosen so there is no gap between the bands. At the FLIP near-field (far-field) site, the ratio of semidiurnal shear variance to near-inertial/diurnal variance is 0.2 (0.1). In the Carter and Gregg spectrum, the ratio is 0.2. The ratio is the same order of magnitude in all three spectra, suggesting it is relevant to shear observations throughout Hawaii. Because $\epsilon$ depends on shear variance squared, the spectra indicate...
<0.5% of the integrated dissipation is associated with the semidiurnal band. This estimate assumes frequencies between inertial and buoyant contribute to the total shear variance. The near-inertial/diurnal shear could result from semidiurnal internal tide energy cascading through frequency space via nonlinear interactions or from wind-generated downward-propagating near-inertial internal waves. Observational evidence from HOME points to a significant contribution from the parametric subharmonic instability (PSI) nonlinear interaction (Rainville and Pinkel 2006; Carter and Gregg 2006). For example, at Hawaii, PSI can transfer energy from the $M_2$ frequency (12.42-h period) to the $M_2/2$ frequency (24.84-h period) and to higher wavenumbers. Over the KC ridge, shear is predominately diurnal, and internal waves with semidiurnal and diurnal frequencies are coupled in the $M_2$ internal tide ray originating from the northern edge of the ridge peak (Carter and Gregg 2006). Diurnal internal wave energy flux in the far field varies with the spring–neap cycle in semidiurnal tidal forcing instead of with diurnal forcing (Rainville and Pinkel 2006). Thus, 0.5% of dissipation due to semidiurnal internal tides is likely an underestimate. However, the inability to determine how much shear in the near-inertial/diurnal frequency band is ultimately from the semidiurnal band precludes a better estimate. We are unable to estimate the fraction of $D_{id}$ due to $M_2$ internal tides.

K06 accounted for along-ridge variations in the dissipation rate by establishing a power law between $\Sigma$, the depth integral of $M_2$ internal tide energy density, and $\Delta$, the depth integral of the dissipation rate: $\Delta \sim \Sigma^{1.5}$. This power law and $M_2$ energy density from the MH02 numerical model are then used to integrate the dissipation rate over regions of the ridge that were not sampled with microstructure shear probes. The Doppler sonar/SeaSoar observations at KC provide further validation for the power law. Energy density $E$ is computed using

$$E = \frac{1}{2} \rho (N^2 \eta^2 + u'^2 + v'^2),$$

where $\eta$ is the isopycnal displacement and $u'$ and $v'$ are the components of perturbation velocity relative to the \{\} mean. The $\Sigma$ and $\Delta$ values are obtained through integration:

$$\Sigma = \left\{ \int_{-330 \text{ m}}^{-100 \text{ m}} E \, dz \right\}$$

and

$$\Delta = \left\{ \int_{-330 \text{ m}}^{-100 \text{ m}} \rho e \, dz \right\}.$$

Energy density can only be computed down to 330 m, because $\eta$ is determined from the SeaSoar data. Variables $\Delta$ and $\Sigma$ are positively correlated (Fig. 7). A linear regression between $\log_{10}(\Delta)$ and $\log_{10}(\Sigma)$ within 100 km of the ridge peak gives power law $\Delta \sim \Sigma^{0.7 \pm 0.2}$. The range of $0.7 \pm 0.2$ is the 95% confidence interval for the linear regression slope. This range falls within the range $1 \pm 0.5$ found by K06. The squared correlation coefficient for the regression is $r^2 = 0.69$. If values out to 170 km from the ridge are included in the regression, $r^2$ decreases to 0.47 and the power to 0.6 ± 0.2.

4. Observations of mixing using potential density inversions

a. Methods

Many instances of decreasing potential density $\sigma_\theta$ with increasing depth occur in the 1-s SeaSoar records. A procedure is required to retain true inversions from among these instances. Our procedure mainly follows that of Ullman et al. (2003) and Galbraith and Kelley (1996). The first step is to isolate pieces of the SeaSoar sawtooth record with monotonic increases or decreases in depth of 300 or more meters (down and up profiles, respectively). Each $\sigma_\theta$ profile is then sorted to obtain the stable profile $\bar{\sigma}_\theta$ with the property that $\bar{\sigma}_\theta$ always increases with depth. If the value $\sigma_\theta(z_i)$ at depth $z_i$ is
assigned to depth \(z_j\) in the sorted profile, its Thorpe displacement is \(d' = z_j - z_p\). Inversions are defined by points with \(d' \neq 0\) between two points with \(d' = 0\). Inversions are checked for the property that the sum of \(d'\) within each inversion equals zero. A number of quantities are then computed for each inversion. The Thorpe scale \(L_T\) is the root-mean-square (rms) Thorpe displacement:

\[
L_T = \left( \frac{d'^2}{N} \right)^{1/2},
\]

where the overbar denotes a mean over all points in each inversion. The inversion height is denoted \(L_Z\). The range of potential density in each inversion is denoted \((\sigma_0)_Z\). The buoyancy frequency assigned to each inversion is \(N\), where \(N\) is computed from \(\sigma_0\). The values \(\overline{\Delta z}\) and \(\overline{\omega}\) are, respectively, the mean depth spacing between successive points and mean SeaSoar vertical speed in each inversion.

Several steps are then used to separate true inversions from spurious ones. As discussed by Ullman et al. (2003), if the slope of the SeaSoar trajectory is less than the slope of an internal wave SeaSoar is passing through, a spurious inversion can result. Internal waves in the presence of no ambient shear become unstable with a wave height to wavelength ratio of greater than 0.1 (Thorpe 1978). A maximum rms internal wave slope in the upper ocean at Hawaii of 0.003 was observed by MRP06. To be cautious, we only accept inversions with mean SeaSoar trajectory slope greater than 0.1. This is accomplished by requiring \(\overline{\omega} > 0.4\, \text{m s}^{-1}\) which follows from a tow speed of 4 m s\(^{-1}\). The tow speed varies between 3.9 and 4.3 m s\(^{-1}\). Virtually all inversions centered at depths between 70 and 320 m have \(\overline{\omega} > 0.4\, \text{m s}^{-1}\). Only inversions with \(L_Z > 2\overline{\Delta z}\) are accepted; this is the minimal Nyquist requirement. A local \(\Delta z\) for each inversion is used because SeaSoar does not have a constant vertical velocity during profiling and thus the vertical spacing of the 1-s data ranges from 0.25 to 3 m. Accepted inversions are also required to have \((\sigma_0)_Z\) greater than the error in \(\sigma_0\). SeaSoar is equipped with two pairs of conductivity and temperature sensors, one pair on each of the lower tail wings. The rms difference in \(\sigma_0\) determined from the two sensor pairs is a measure of error in \(\sigma_0\). Below the mixed layer, this error is typically \(1 - 3 \times 10^{-3}\, \text{kg m}^{-3}\). All accepted inversions have \((\sigma_0)_Z > 3 \times 10^{-3}\, \text{kg m}^{-3}\). Of 64 703 inversions initially identified, the slope, \(L_Z\), and \((\sigma_0)_Z\) requirements leave only 2301 as accepted inversions. The run-length test outlined by Galbraith and Kelley (1996) is then applied to the remaining inversions. A “run” is a set of consecutive \(d'\) values with the same sign. For each inversion, the maximum run length \(r\), the number of values in the longest run, is recorded. The PDF of \(r\) from all inversions is compared with the theoretical PDF of run-length noise for which positive and negative \(d'\) are equally likely. For run lengths greater than or equal to 4, the PDF of \(r\) is 2 times the noise PDF. Inversions with \(r \geq 4\) are accepted, leaving 906 inversions. About 20% of these inversions are then rejected using the watermass test of Galbraith and Kelley (1996), which requires a tight relationship between potential temperature \(\theta\) and salinity \(S\). After visual inspection of the \(\theta-S\) relationship for inversions, we choose watermass-test criteria less strict than those of Galbraith and Kelley (1996) by a factor of 2. The result of the overall procedure is 297 inversions accepted below the mixed layer. Observations of inversions with a towed profiler are certainly of lower quality than those obtained by vertical profiling. However, turbulent patches in the ocean are horizontally elongated with a typical vertical-to-horizontal dimension aspect ratio of order 0.01-0.1 (Gregg 1987). Because the SeaSoar trajectory slope for retained inversions always exceeds 0.1, inversion evidence of turbulent mixing is within the scope of our observations.

b. Results for accepted inversions

Four inversions with \(L_Z > 25\) m are shown in Fig. 8 along with the sorted \(\sigma_0\) profiles. Each of these inversions appears in the full-depth \(\sigma_0\) profile with [\(\partial \sigma_0 / \partial z\)] clearly smaller than values just outside of the inversion.
One inversion west of Nihoa Island in the first along-ridge survey extends 39.6 m in depth and includes several subinversions (Fig. 8a). An inversion south of Nihoa Island in the second along-ridge survey (Fig. 8b) and another 42 km from the ridge at the Kauai Channel (Fig. 8c), both have tall Z shapes. Near Kaula Island, a profile of \( \sigma_z \) exhibits instability over a 45-m depth range (Fig. 8d). The inversions in Fig. 8 are extreme cases. Over all inversions, mean \( L_Z \) is 9 m and mean \( L_T \) is 4 m. The range of \( N \) is 1 to \( 16 \times 10^{-3} \) rad s\(^{-1} \) and mean \( N \) is \( 3 \times 10^{-3} \) rad s\(^{-1} \). Most inversions with \( N > 4 \times 10^{-3} \) rad s\(^{-1} \) are near the mixed layer base. The deepest inversions occur near 350-m depth where typical values of the along-track mean buoyancy frequency used to compute \( \epsilon_{IW1} \) are \( > 6 \times 10^{-5} \) rad s\(^{-1} \). This buoyancy frequency increases with depth above 350 m. Inversions beneath the mixed layer base have a local buoyancy frequency less than the smallest observed along-track mean values.

Dillon (1982) found an empirical linear relationship between the Thorpe scale \( L_T \) and the Ozmidov scale \( L_O = (\epsilon / N^3)^{1/2} \). This relationship has been regularly used in the literature to obtain an inversion-based estimate of the dissipation rate:

\[
e_\tau = 0.64 L_T^2 N^3. \tag{7}
\]

However, this relationship is contingent upon the gradient Richardson number (Ri) being approximately constant, which is not true of our observations. If the Ri dependency is retained in Dillon’s derivation, the inversion-based estimate of dissipation becomes

\[
e_\tau = \frac{aL_T^2N^3}{Ri^{3/2}}, \tag{8}
\]

where \( a \) is a constant and \( Ri = \frac{N^3}{(dU/dz)^2} \), where \( dU/dz \) is the mean shear over each inversion. The Ri ranges from \( 10^{-2} \) to \( 10^2 \). Scatterplots of \( e_\tau \) and \( e_{IW1} \) versus \( e_{IW1} \), the estimate of the dissipation rate from the parameterization (section 3), are presented in Figs. 9–10. The \( e_{IW1} \) values appearing in the figures are means over each inversion. There is no correlation between \( e_\tau \) and \( e_{IW1} \). If the gradient Richardson number dependence is retained, the correlation improves (Fig. 10). However, this occurs because \( e_{IW1} \) depends on shear to the fourth power and \( e_\tau \) on shear to the third power. Thus, the improved inversion-based estimate of the dissipation rate, \( e'_\tau \), is not independent from \( e_{IW1} \). The constant \( a = 6.6 \times 10^{-3} \) used in Fig. 10 is determined by minimizing the rms difference between \( \log_{10}(e'_\tau) \) and \( \log_{10}(e_{IW1}) \).

Figures 9–10 highlight that one should not expect \( e_\tau \) and \( e_{IW1} \) to agree when variation in Ri is large.

The main value of our observations is that they allow examination of the spatial distribution of inversions. Mean \( L_Z \) is displayed on a map to show this spatial distribution and changes in inversion size around the ridge (Fig. 11). The mean is over all inversions below
100-m depth that fall within the boxes defined in section 3b. Most displayed boxes contain only one inversion, but one box centered at (23.15°N, 163.11°W) contains 14 and several near the ridge in the eastern KC contain more than 5. Accepted inversions occur most frequently near the main ridge axis. This can be seen by comparing the boxes displayed in Fig. 11 with all available boxes shown in Fig. 5 or the cruise track shown in the background. The inversions congregate near the ridge at FFS/BB, west of and near Nihoa Island, around Kaula, Niihau, and Kauai Islands between 160.6° and 159.4°W, and in the eastern KC. Inversions tend to be absent farther from the ridge. There are virtually none on the northern side of the KC, a region that was well sampled. More are found far from the ridge at FFS/BB and where sampling was heaviest on the south side of KC. Inversions are not found at the gaps in the ridge between 162° and 160°W, nor at the Necker Island section of ridge centered at 164.5°W. It is difficult to discern clear along-ridge patterns in $L_z$, a result of having such a small number of inversions in any one box. However, consistently large mean $L_z$ is found east of FFS and west of Nihoa Island.

Observed inversions are collocated with large $K_\nu(e_{IW1})$ (Figs. 5 and 11). A comparison can be made between the PDF of $K_\nu(e_{IW1})$ inside and outside of accepted inversions (Fig. 12). The $K_\nu$ is used instead of $\epsilon_{IW1}$ to reduce effects of depth dependence since $K_\nu$ has less depth dependence in the upper ocean than $\epsilon$ (Lee et al. 2006; K06). The grid point of each $K_\nu(e_{IW1})$ value is considered inside an inversion if it is the closest point in time to that inversion and if it falls within the depth range of the inversion. Values of order $10^{-4}$ and $10^{-3}$ m$^2$ s$^{-1}$ occur more frequently inside than outside inversions. The inside mean is $2.5 \times 10^{-4}$ m$^2$ s$^{-1}$ and the outside mean is $1.5 \times 10^{-3}$ m$^2$ s$^{-1}$, a relative difference of 45%. The inside median is $2.2 \times 10^{-5}$ m$^2$ s$^{-1}$ and the outside median is $1.1 \times 10^{-3}$ m$^2$ s$^{-1}$, a relative difference of 68%. Where inversions large enough to be resolved occur, mixing predicted from internal wave shear ($K_\nu$) is on average larger.

5. Summary and conclusions

The goal of this study was to quantify turbulent dissipation ($\epsilon$) and mixing ($K_\nu$) and investigate their spa-
Fig. 12. PDFs of \( \log_{10}(K_r) \) outside and inside density inversions in the depth range from 100 m to the bottom of the SeaSoar profiles. The range of each bin includes all values that would be considered the same order of magnitude. For example, the range of the fourth bin is \( 5 \times 10^{-8} \) to \( 5 \times 10^{-5} \) m\(^2\) s\(^{-1}\). However, the first bin contains contributions from a long tail extending several orders of magnitude lower. Values of order \( 10^{-4} \) and \( 10^{-3} \) m\(^2\) s\(^{-1}\) occur more frequently inside inversions than outside inversions.

The relative contributions of \( M_2 \) internal tides, internal tides of other periods, and wind-generated near-inertial internal waves to observed shear cannot be determined from the Doppler sonar survey. However, a few observations point to the importance of internal tides to mixing at the Hawaiian Ridge: 1) The across-ridge decay of \( K_r \) at KC and south of FFS/BB indicates the ridge is a source of shear. Wind-generated shear is not anticipated to increase over the ridge. 2) The \( K_r \) is enhanced along ridge where \( M_2 \) energy density in the MH02 model results is enhanced. 3) At one location in the eastern KC between 200- and 450-m depth and over the 1000-m isobath, typical values of \( K_r \) are two or more orders of magnitude larger at the peak of spring tide than at the end of neap tide. This change occurs immediately above the \( M_2 \) internal tide ray of largest kinetic energy density emanating from the ridge (MRP06). We speculate that \( M_2 \) internal tides become unstable within or near the ray, dissipating in part at an across-ridge position just south of the ridge peak.

Because it was necessary to infer \( \varepsilon \) indirectly, results and conclusions concerning the magnitude of the dissipation rate and eddy diffusivity are less certain than those concerning their spatial structure. However, confidence in the accuracy of \( K_r \) inferred through the Gregg (1989) parameterization is established through comparisons with microstructure profiler data from HOME in the appendix. On average, \( K_r \) from the parameterization agrees with that from the microstructure profilers to within a factor of 2. All values of \( K_r \) shown in Figs. 3–4 computed with the parameterization are greater than the canonical open-ocean value of \( 10^{-5} \).
m² s⁻¹. Maximum values are greater than 10⁻² m² s⁻¹. In the upper ocean, we have addressed a very basic objective of the HOME project by learning that $K_p$ is enhanced near the Hawaiian Ridge, supporting the results of others (K06). An upper limit on the integrated dissipation rate from $M_2$ internal tides to turbulence within 50 km of the Hawaiian Ridge (near the 4000-m isobath at the internal tide generation sites) is very roughly estimated to be $6.5 \pm 0.5$ GW. Frequency spectra of shear obtained during HOME from FLIP- and ship-based Doppler sonar indicate the semidiurnal frequency band contributes less than 0.5% of the integrated dissipation rate. The shear spectra have clear maxima in the near-inertial/diurnal band, but the source of shear in this band is a major unknown. Initial evidence points to a significant transfer of energy from semidiurnal internal tides to diurnal internal waves through nonlinear interactions (Carter and Gregg 2006; Rainville and Pinkel 2006). However, wind-generated near-inertial internal waves cannot be ruled out as a source for the shear we observe. The $6.5 \pm 0.5$ GW upper limit on the integrated dissipation rate can be compared to the predicted $7.10^{15.5}$ GW $M_2$ internal tide dissipation rate within the 4000-m isobath (barotropic loss minus internal tides escaping 4000-m isobath). These values could be used in a discussion about the budget of energy transfer from barotropic to internal tides to turbulent mixing at the Hawaiian Ridge. However, careful consideration should be given to the values’ errors and definitions. A conclusion suggested by both the across- and along-ridge spatial structure and the integrated dissipation rate is substantial dissipation of internal tides occurs close to the geographic feature where they are generated.

Our observations of density inversions provide direct evidence of large mixing events occurring near the Hawaiian Ridge. Extreme cases have vertical dimension on the order of tens of meters. The spatial distribution of these large mixing events supports the results obtained from the estimates of $K_p$; mixing events are largest near the ridge and at certain locations along the ridge. Resolved inversions occur mostly near the main ridge axis and especially at FFS/BB, west of and at Nihoa Island, around the islands of Kauai, Niihau, and Kaula, and in the eastern KC. Inversions tend to be absent far from the ridge and at gaps in the ridge. The difference in average $K_p$ inside and outside of inversions indicates that where large mixing events occur internal wave shears are enhanced.

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APPENDIX

Estimating the Turbulent Dissipation Rate of Kinetic Energy

The first purpose of this appendix is to discuss procedures for computing three estimates of the turbulent dissipation rate of kinetic energy from the SeaSoar/ Doppler sonar (SS/DS) data. Estimate $\varepsilon_{IW1}$ is obtained using the procedure of Gregg (1989) and is proportional to the fourth power of shear computed from first differences. Estimate $\varepsilon_{IW2}$, providing an average value over the upper ocean and over some horizontal region, is obtained from vertical wavenumber spectra of shear using a procedure described by Gregg et al. (2003) but without the shear-to-strain ratio and latitude-dependent factors. Estimate $\varepsilon_{GM}$ is $\varepsilon_{GW}$ multiplied by the shear-to-strain ratio and latitude-dependent factors. The second purpose of this appendix is to check the accuracy of the dissipation rate estimates and determine which estimate is best to use in the main text by making comparisons with microstructure measurements from HOME.

The $\varepsilon_{IW1}$ estimate (W kg⁻¹) is computed using

$$\varepsilon_{IW1} = 6.73 \times 10^{-10} \left( \frac{N}{N_0} \right)^2 \left( \frac{S_{OBS}^2}{S_{GM}^2} \right)^2,$$

(A1)

with $N_0 = 5.24 \times 10^{-3}$ rad s⁻¹ (Gregg 1989). Equation (A1) applies to the thermocline. The mixed layer depth during the survey was typically 50–75 m. We compute $\varepsilon_{IW1}$ only below 100-m depth. The $N$ is the buoyancy frequency, obtained by smoothing local SeaSoar-derived buoyancy frequency squared. The smoothing is over 11 consecutive profiles, a horizontal range of approximately 40 km. SeaSoar consistently reached a depth of 330 m. Useful data from the 50-kHz sonar extends to 550 m, so below 330 m we use an exponential fit to $N$ determined from the average of three full-depth Hawaii Ocean Time Series (HOTS) profiles taken north of the Kauai Channel near the time of the survey. Here $S_{GM}^2$ is defined by

$$S_{GM}^2 = \int_0^{k_{GM}} \phi_{GM}(k) \, dk,$$  \hspace{1cm} (A2)
where $\phi^\text{GM}_S(k)$ is the Garrett–Munk GM76 (Cairns and Williams 1976) vertical wavenumber shear spectrum according to Gregg and Kunze (1991):

$$\phi^\text{GM}_S(k) = \frac{12\pi^2Eb^3N_0}{J_\nu} \frac{k^2}{(1 + k/k_c)^2}. \quad (A3)$$

The cyclic vertical wavenumber is $k$, and $k_c^\text{GM} = 0.1$ cycles m$^{-1}$ is the cutoff wavenumber. The GM parameters are $E = 6.3 \times 10^{-5}$, $b = 1300$ m, and $J_\nu = 3$, and $k_S = (J_\nu N)/(2b N_0)$. Thus, $S^\text{GM}$ is the total variance of shear contained in all wavenumbers less than $k_c^\text{GM}$. Functionally, $S^\text{GM}$ varies from 0.725$N^2$ at $N = 1 \times 10^{-4}$ rad s$^{-1}$ to 0.625$N^2$ at $N = 1 \times 10^{-2}$ rad s$^{-1}$. Observed $N$ ranges from $1.2 \times 10^{-2}$ rad s$^{-1}$ at 100 m to $5 \times 10^{-3}$ rad s$^{-1}$ at 550 m. The quantity $S^\text{OBS}_S(s^{-2})$ is the observed equivalent of $S^\text{GM}$, computed as the squared magnitude of first-difference shear multiplied by a correlation factor $c$:

$$S^\text{OBS}_S = c[(\Delta u/\Delta z)^2 + (\Delta v/\Delta z)^2]. \quad (A4)$$

The difference interval $\Delta z$ is the vertical grid spacing of 6.15 m discussed in section 2, and $u$ and $v$ are, respectively, the east and north components of velocity from the Doppler sonar. First-differencing and Doppler sonar range-averaging both attenuate higher-wavenumber shear variance. The correlation factor compensates for this loss of variance and is computed using

$$c = \frac{S^\text{GM}}{\int_{k_c}^{k_c^\text{GM}} \phi^\text{GM}_S(k)|F_{RA}(k)|^2 \operatorname{sinc}^2(k \Delta z) dk} \quad (A5)$$

following Wijesekera et al. (1993) and Gregg and Sanford (1988). Equation (A5) assumes the observed shear spectrum has the same shape as the GM shear spectrum. The $\operatorname{sinc}^2$ factor is result of first differencing. Over any listening interval, a Doppler sonar echo is a weighted mean of reflections from a range of depths. For the Reveille Doppler sonar, the weighting is trapezoidal with approximately two-thirds of the data used in the mean for one bin contributing to the mean in the next bin. The range-averaging filter is given by

$$F_{RA}(k) = \frac{\{\cos[a(Q + 1)k] - \cos[a(3Q - 1)k]\}}{(2ak)^2(Q - 1)} \quad (A6)$$

with $a = \pi c_c \cos(\pi/6)$. For the 50-kHz sonar, $Q = 11$, $\tau = 4.8 \times 10^{-4}$ s, and $c_c = 1480$ m s$^{-1}$ is the speed of sound. The correction factor varies from $c = 3.9$ at 550 m to $c = 4.4$ at 100 m. Here the curly braces have no special significance.

A second procedure to estimate the turbulent dissipation rate (W kg$^{-1}$) was developed (Kunze et al. 1992; Polzin et al. 1995; Kunze and Sanford 1996; Gregg et al. 2003) in response to the criticism of Gargett (1990) that Gregg (1989) did not consider observed shear spectra with cutoff vertical wavenumber different from the cutoff in GM. We use the procedure as it is presented in Gregg et al. (2003). The second estimate is given by

$$\varepsilon_{tw2} = 6.73 \times 10^{-10} \left(\frac{N}{N_0}\right)^2 \left(\frac{\phi^\text{OBS}_S}{\phi^\text{GM}_S}\right)^2, \quad (A7)$$

where

$$\langle \phi^\text{OBS}_S \rangle = \frac{1}{(k_c^\text{OBS} - k_{min})} \int_{k_{min}}^{k_c^\text{OBS}} \phi^\text{OBS}_S(k) dk \quad (A8)$$

is the mean of the observed vertical wavenumber shear spectrum from the lowest resolved wavenumber $k_{min}$ to a cutoff wavenumber $k_c^\text{OBS}$ and

$$\langle \phi^\text{GM}_S \rangle = \frac{S^\text{GM}}{(k_c^\text{GM} - k_{min})}. \quad (A9)$$

Here $k_c^\text{OBS}$ is defined by

$$\int_{k_{min}}^{k_c^\text{OBS}} \phi^\text{OBS}_S(k) dk = S^\text{GM}. \quad (A10)$$

That is, $k_c^\text{OBS}$ is the wavenumber at which the variance in the observed spectrum equals the variance in the GM spectrum up to its rolloff wavenumber. The inclusion of the contribution to $S^\text{GM}$ from interval 0 to $k_{min}$ causes negligible error as it is less than 0.5% of the total $S^\text{GM}$. Combining Eqs. (A7)–(A10) results in

$$\varepsilon_{tw2} = 6.73 \times 10^{-10} \left(\frac{N}{N_0}\right)^2 \left(\frac{k_c^\text{GM} - k_{min}}{k_c^\text{OBS} - k_{min}}\right)^2. \quad (A11)$$

The procedure for computing the observed shear spectrum $\phi^\text{OBS}_S(k)$ to obtain $k_c^\text{OBS}$ is as follows. A vertical trend is removed from each $u$ and $v$ profile and the vertical wavenumber periodogram computed for each. Each periodogram is divided by $|F_{RA}(k)|^2$ to correct for range-averaging. The velocity component spectra $\phi^\text{OBS}_u(k)$ and $\phi^\text{OBS}_v(k)$ are then computed by applying a horizontal mean to the periodograms. (For Fig. A1 the horizontal mean is over profiles taken within certain isobath limits and for Fig. A2 the mean is over all profiles that fall within bins 10 km wide in across-ridge position.) The shear spectra are determined using

$$\phi^\text{OBS}_S(k) = (2\pi k)^3[\phi^\text{OBS}_u(k) + \phi^\text{OBS}_v(k)]. \quad (A12)$$
The shear spectra are then integrated to determine $k_{OBS}^c$, the wavenumber at which the integral equals $S^2_{GM}$. The $N$ used in calculating $\varepsilon_{IW2}$ is from the vertical mean of the $N^2$ used for $\varepsilon_{IW1}$. Gregg et al. (2003) apply two factors to $\varepsilon_{IW2}$ to determine their full definition of the dissipation rate:

$$\varepsilon_{GSW} = \varepsilon_{IW2}G(R)L(\theta, N), \quad (A13)$$

The factor $G(R)$ is a result from Polzin et al. (1995) accounting for observations made in an internal wave field with frequency spectra of non-GM shape and is given by

$$G(R) = \left(1 + \frac{1}{4R} - \frac{2}{R - 1}\right)^{1/2}. \quad (A14)$$

where

$$R = \frac{1}{N^2} \int_0^{\infty} \varepsilon_{OBS}^S \phi_S^{OBS}(k) \, dk$$

$$= \frac{1}{N^2} \int_0^{\infty} \varepsilon_{OBS}^{\eta} \phi^{\eta}_{OBS}(k) \, dk. \quad (A15)$$

The denominator is the strain $(\eta)$ variance where $\eta$ is the isopycnal displacement. For our data, $\eta$ is a perturbation from the same horizontal mean used in computing the velocity component spectra. The strain spectrum is computed in the same way as the shear spectra. Factor $G(R)$ is equal to 1 when $R$ has the GM value of 3. The latitude $(\theta)$ dependence factor is given by

$$L(\theta, N) = \frac{f(\theta)}{f(30^\circ)} \cosh^{-1}\left(\frac{N}{f(\theta)}\right) \cosh^{-1}\left(\frac{N_0}{f(30^\circ)}\right)^{-1}. \quad (A16)$$

Across observations from a number sites, Gregg et al. (2003) find that $\varepsilon_{IW2}$ has the largest range and is the most important among the factors in Eq. (A13).

To check the accuracy of $\varepsilon_{IW1}$, $\varepsilon_{IW2}$, and $\varepsilon_{GSW}$ and decide which estimate should be used in the main text, average $K_p$ derived from the estimates is compared to average $K_p$ derived from microstructure measurements made during HOME. All comparisons are with results in the K06 article where details about the microstructure-measuring instruments can be found. A comparison is made with average $K_p$ from the Absolute Velocit Profiler (AVP) and the Advanced Microstructure Profiler (AMP) (Fig. A1). Both instruments are equipped with shear probes. AVP is untethered and was dropped at KC and FFS/BB at the 3000-m isobath. Average AVP profiles of $K_p$ as a function of height.
above bottom (HAB) are obtained by averaging over all data between the 2500- and 3500-m isobaths and west of Oahu). The SS/DS average for FFS/BB is in terms of order of magnitude, perhaps a better measure of how well the two datasets agree is the mean of the ratio \( R1, 0.1 \) 1.5 1.3 \( K_{micro}^{SS/DS} \) for the values shown in Fig. A1. Means \( (\langle \rangle) \) are computed over the range of the Doppler sonar data (R2, 100–550-m depth) and the range of the SeaSoar data (R1, 100–330-m depth). The rms difference and mean ratio are measures of how well each of the estimates of \( K_p \) derived from the SS/DS data compare with \( K_p \) derived from the microstructure profiler measurements.

<table>
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<th>Microstructure profile</th>
<th>SS/DS estimate</th>
<th>( \langle k_{micro}^{SS/DS} - k_{micro}^{SS/DS} \rangle^{1/2} ) ((10^{-3} m^2 s^{-1}) )</th>
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Fig. A2. Comparison between the average across-ridge structure of \( K_p \) for the depth range 100–550 m obtained from 1) K06’s structure function (see text) and 2) the SS/DS data. Averages are over all across-ridge sections at (a) KC and (b) FFS/BB and (c) over the entire survey. Here \( \langle K_p(s\text{, struct. func.}) \rangle_a \) is evaluated at each survey position and depth and then averaged to produce the curves in (a)–(c). The \( \langle \rangle \) operator denotes a mean over bins 10 km wide in across-ridge position, and \( \langle \rangle_a \) denotes a mean over 100–550-m depth. At KC and FFS/BB, across-ridge position is determined using K06’s coordinate systems (defined in their Fig. 2).

Kaena Ridge where bottom depths are generally less than 1500 m (Fig. A1c). Here \( K_p \) has also been averaged into 50-m depth bins. The corresponding average for the SS/DS-derived estimates is over all data taken in the KC region shallower than the 1500-m isobath.

Two measures are used to determine which SS/DS-derived estimate of \( K_p \) agrees best with the microstructure profiler results. The first measure is the root-mean-square (rms) difference between the SS/DS-derived \( K_p^{SS/DS} \) and microstructure-derived \( K_p^{micro} \) values in Fig. A1. The second measure is the mean of the ratio \( K_p^{micro}/K_p^{SS/DS} \). The results are in Table A1. For the R2 range, the rms difference indicates the KC AVP profile agrees better with the \( K_p(e_{IW1}) \) estimate, the FFS/BB AVP profile agrees better with the \( K_p(e_{IW2}) \) estimate, and that neither estimate is clearly better for the AMP profile comparison. Since \( K_p \) levels tend to be discussed in terms of order of magnitude, perhaps a better measure of how well the two datasets agree is the mean of the ratio \( K_p^{micro}/K_p^{SS/DS} \). For example, consider the case in which typical values over the R2 range are \( K_p^{micro} = 1 \times 10^{-4} m^2 s^{-1}, K_p^{SS/DS} = 2 \times 10^{-4} m^2 s^{-1} \), and \( K_p^{micro} = 2 \times 10^{-5} m^2 s^{-1} \) (similar to the case for AVP at FFS/BB). The rms difference between \( K_p(e_{IW1}) \) and \( K_p^{micro} \) will be less than that between \( K_p(e_{IW1}) \) and \( K_p^{SS/DS} \), even though \( K_p(e_{IW2}) \) is a factor of 5 less than \( K_p^{micro} \) and \( K_p(e_{IW1}) \) is only a factor of 2 greater than \( K_p^{micro} \). Based on the ratio measure, \( K_p^{micro} \) agrees better with \( K_p(e_{IW1}) \) over the R2 range. The agreement is less
than a factor of 2 with each of the microstructure profiles. In the R1 range, by the ratio measure the AVP profiles agree best with the $K_p(e_{GW1})$ estimate, but no estimate is clearly best for the AMP values. The SS/DS measurements do not extend deep enough to test the vertical dependence of K06’s structure function [Eq. (A18)]. However, in the AMP comparison $K_p(e_{GW1})$ increases somewhat with increasing depth.

A comparison is also made between the across-ridge structure of $K_p$ estimated from the SS/DS data and that predicted by K06’s structure function (Fig. A2). The structure function is

$$K_p(x, h) = K_p(x/L)\tilde{\zeta}(h/H),$$

where $x$ is the across-ridge position relative to the center of the coordinate system, $L = 1$ km, $h$ is the bottom depth, and $K_p = 10^{-5}$ m$^2$ s$^{-1}$. The vertical part of the function is

$$\tilde{\zeta}(h/H) = \begin{cases} 1 & h/H > 0.55 \\ 10^{-1.18[1 - (h/H)^0.55]} & h/H < 0.55 \end{cases}$$

(A18)

The across-ridge part of the function is given by the set of values:

$$x = [-60, -10, 0, 7, 14, 24, 74] \text{ km} \log_{10} x = [0, 1, 1.3, 1.3, 1, 0].$$

(A19)

The across-ridge structure ($\chi$) is symmetric about $x = 7$ km and is flat out to $x = 0$ and 14 km. This is followed by a steep drop out to $x = -10$ and 24 km and then a more gentle drop to $x = -60$ and 74 km. At KC, $\langle K_p(e_{GW1})\rangle_d$ is no more than twice $\langle K_p(\text{struct. func.})\rangle_d$ from positions 20 km to 40 km (-40 to 50 km) (Fig. A2a). At FFS/BB, SS/DS-derived $K_p$ is well above $\langle K_p(\text{struct. func.})\rangle_d$ except from positions 20 to 30 km (Fig. A2b). When all survey data are averaged, $\langle K_p(e_{GW1})\rangle_d$ and $\langle K_p(e_{GW2})\rangle_d$ are no more than twice $\langle K_p(\text{struct. func.})\rangle_d$ from positions 0 to 40 km (-20 to 40 km) (Fig. A2c). Farther from the ridge peak, across-ridge gradients of $\langle K_p(e_{GW1})\rangle_d$ and $\langle K_p(e_{GW2})\rangle_d$ are smaller than gradients of $\langle K_p(\text{struct. func.})\rangle_d$. This discrepancy could result from the vertical gap between the SS/DS data and those used in establishing the across-ridge part of the structure function. Function $\chi(x/L)$ was established using microscale shear data obtained by towing the instrument Marlin at depths of 700 km and 900 m at KC and 500, 1000, and 1500 m at FFS. The across-ridge gradient of $K_p$ at KC in the upper 550 m could be less than the gradient at depths below 550 m, but this effect is not included in the structure function. Another possible reason for the discrepancy between gradients is that while Eq. (A1) successfully gives $K_p \sim 10^{-4}$ m$^2$ s$^{-1}$ near the AVP and AMP profiles, Eq. (A1) does not depend correctly on shear. The power of shear in Eq. (A1) would have to be increased to move the gradient closer to that predicted by the structure function. The changing ratio between $\langle K_p(e_{GW1})\rangle_d$ and $\langle K_p(e_{GW2})\rangle_d$ is a consequence of changes in shear spectra with across-ridge position. For example, on the south side of KC, the shear spectra at positions -50 and -40 km have greater $k^{GM}$ but lesser shear variance between wave numbers 0 and $k^{GM}$ than spectra at neighboring positions where the $K_p$ discrepancy is smaller. Such spectral variations are what the $e_{GW2}$ version of the parameterization addresses. The across-ridge structure comparison does not clearly demonstrate whether $K_p(e_{GW1})$ or $K_p(e_{GW2})$ is a better estimate to use.

Supported by agreement with the microstructure profiler data, we use the Gregg (1989) estimate ($e_{GW1}$) in section 3. The agreement between $\epsilon$ derived from microscale shear and $\epsilon$ estimated using the parameterization can also be checked within the AVP dataset, as the AVP also measures the meter-scale velocity field. Doing so, Lee et al. (2006) find AVP $\epsilon$ from the HOME survey is consistent with the Gregg (1989) parameterization within a factor of 2 over the range $5 \times 10^{-10}$ to $10^{-7}$ W kg$^{-1}$.

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