CORRESPONDENCE

Comments on “The Interaction of an Eastward-Flowing Current and an Island: Sub- and Supercritical Flow”

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ABSTRACT

In regards to the recent paper by Pedlosky and Spall (Journal of Physical Oceanography, November 2015), this comment maintains that the steady-state solutions of Rossby waves in a uniform eastward current past an island have waves on the upstream side that are not caused by the island because of inappropriate boundary conditions and the assumed form of the solution. The solutions are interesting but are not the solutions to the problem as posed in their paper. Similar upstream waves in a time-dependent numerical model are also inconsistent with linear Rossby wave theory, though the reasons for their presence are uncertain.

The Pedlosky and Spall paper (Pedlosky and Spall 2015) consists of two parts. The first part (their sections 2–4) describes an analytical model of steady eastward flow past an obstacle (a vertical barrier, aligned north–south) on a classic beta ($\beta$)-plane of uniform depth in a zonal channel with northern and southern boundaries aligned east–west (see their Fig. 2). The model is linear (but complete because the model involves flow between two rigid, horizontal, flat plates), and the authors describe the linear Rossby wave motions forced by flow past the obstacle. In situations where waves occur in their analysis, waves are obtained upstream of the obstacle as well as downstream (Pedlosky and Spall 2015, their Figs. 6, 7, 8, and 9). We believe that these wave patterns are spurious and unphysical, for reasons given below. The second part (their section 5) consists of a nonlinear, time-dependent numerical model producing some flows with similar features; this model is discussed subsequently.

The dynamics of Rossby waves forced by an obstacle in relative motion through a fluid were studied in the 1960s [in particular, by Lighthill (1966, 1967, 1978)] as part of a general study of waves in dispersive systems. A key to the properties of such waves is their dispersion relation. For a given system, such as waves on a beta plane forced by an obstacle moving westward at velocity $U$ (or, equivalently, eastward flow past a stationary obstacle), important information is given by the dispersion relation. For steady-plane Rossby waves of the form $\psi \propto e^{i(kx+ly)}$, where $\psi$ denotes a perturbation streamfunction, $x$ and $y$ are the eastward and northward coordinates, and $k$ and $l$ are wavenumbers, the dispersion relation takes the form

$$-Uk(k^2 + l^2) + \beta k = 0$$

in a system consisting of a single layer of uniform depth. A plot of this relation (sometimes known as a “Lighthill diagram”) is shown in Fig. 1 [which is effectively the same as Fig. 1 of Lighthill (1967)]. It consists of the circle $k^2 + l^2 = \beta/U$, and the axis $l = 0$, marked as solid lines. Associated with these lines are arrows emanating from them, which denote the directions of the group velocity $[c_g = \nabla \psi(\omega)]$ of the waves for each value of $(k, l)$. These show the directions of the group velocity in physical space, where the $k$ axis is identified with the $x$ axis, and the $l$ axis is defined with the $y$ axis. Waves on the circle with wavenumber of magnitude $(\beta/U)^{1/2}$ all have group velocity with rightward (eastward) components so that the waves are all on the lee side. Waves with the wavenumber on the $l$ axis, on the other hand, denote columnar motions and have leftward (westward) components of group velocity inside the circle and rightward components outside; this implies that larger-scale columnar motions (associated...
with blocking) may be found upstream and smaller-scale ones may be found downstream. This is for an unbounded environment, but the additional presence of north and south boundaries reflects the waves to give (sinusoidal) modes in the channel, which must be located downstream, with no waves upstream.

To solve the appropriate steady-state Helmholtz equation, therefore, the correct approach is to require no waves upstream (other than motions with \( k = 0 \)), and waves on the downstream side must conform to a radiation condition, implying no incoming waves. This procedure has been tested experimentally by White (1971; in the very first volume of the Journal of Physical Oceanography), who compared a steady-state model based on the radiation condition embodied in the linear dispersion relations described above, with relevant laboratory experiments. In his corresponding experiments, no upstream waves were observed and remarkably good agreement was obtained between the model and observations, even for lee waves of large amplitude.

For the Pedlosky and Spall (2015) analysis, their downstream condition does not have the form of a radiation condition but instead specifies that the flow at the exit boundary be uniform [in “most cases”; see their paragraph following Eq. (2.12)]. This condition conserves mass flux but is not a radiation condition, since the waves have a sinusoidal north–south structure and not a uniform one. On the upstream side, it is stated that the flow at the western entrance \( x = x_w \) is “independent of the cross-channel coordinate \( y \)” (Pedlosky and Spall 2015, p. 2808), but boundary condition (2.6b) does not specify this. In fact, the solution procedure in their section 2b results in a northward (or north–south) velocity at \( x = x_w \) that is a function of \( y \) for the flow depicted in Figs. 6–9 of Pedlosky and Spall (2015) and which can be regarded as the source of the waves on the upstream side of the island in these figures. For the correct solution, the terms on the upstream side should be exponential in \( x \) or possibly independent of it (giving columnar modes) if \( f \) is sufficiently small (specifically, this requires \( j \pi < b^{1/2} \) in the authors’ notation).

We should point out that there is a close analogy here with uniformly stratified flow over obstacles (see, e.g., section 5.3 of Baines 1995, 1998), particularly when the obstacle/island has small height/width. Columnar modes embodying upstream blocking appear when the obstacle height is large, as the island width is in Figs. 5–9 of Pedlosky and Spall (2015).

The second part of their paper (their section 5) describes results from a time-dependent numerical model that has similar geometry, which also shows stationary waves upstream (Pedlosky and Spall 2015, their Figs. 1, 11, 12, 14). This model is a reentry channel, with damping at the upstream end to prevent disturbances recirculating. It is nonlinear, on a \( \beta \) plane, but with the additional feature of describing motion on a shallow layer of variable depth \( h \), overlying an infinitely thick, motionless deep layer. Results from this model are shown in Figs. 1 and 10–14 of Pedlosky and Spall (2015). Figures 1 and 11–14 of Pedlosky and Spall (2015) show upstream waves that resemble those in the steady-state model described above. Can these upstream waves be understood in terms of linear wave theory?

This model is time dependent, and an appropriate analysis should reflect this. Could these upstream waves be long-lived transients? The standard dispersion relation for long, zonally traveling Rossby waves \( \omega \propto \sin(ly)e^{ik(x-wt)} \) (with frequency \( \omega \)) in this model has the form

\[
\omega = -\beta k (k^2 + f^2 + f^2/c^2) + U k, \quad (2)
\]

where \( f \) is the mean Coriolis frequency, and \( c = (g' \bar{h})^{1/2} \) is the gravity long-wave speed (where \( \bar{h} \) is the mean layer thickness, and \( g' \) is the reduced gravity). For steady flow (\( \omega = 0 \)), the dispersion relation is similar to that shown in Fig. 1, and an example of it for small \( \omega \) is shown in Fig. 2. For sufficiently small values of both current speed \( U \) and \( \omega \), the equation has two solutions for the zonal wavenumber \( k \) and hence has two sets of lee/upstream waves (as in Fig. 1). The group velocity of the shorter waves is to the east and gives the lee waves as described above. The group velocity for the long waves [small \( K = k(U/\beta)^{1/2} \) is to the west and has the potential to generate stationary waves to the west in front of the island. However, while these waves may have a meridional structure, they are extremely long zonally (\( K \ll 1 \). As
such they do not resemble the waves in Figs. 1, 11, 12, and 14 of Pedlosky and Spall (2015).

We note that in each of Figs. 1, 11, and 12 of Pedlosky and Spall (2015), the upstream waves have the same phase in relation to the location of the upstream boundary, where the amplitude of these upstream waves is largest. This suggests that the upstream boundary has some reflective (or generative) properties for these upstream waves. We would point out that in the studies by Tansley and Marshall (2001), with a very similar model, upstream waves are absent from their steady simulations except for their Fig. 3b, which shows an upstream wave with a similar phase configuration to the upstream boundary as those shown by Pedlosky and Spall (2015).

In the study by White (1971), no upstream or downstream damping was employed in his model. We also note that no upstream waves were seen in the numerical study by Page and Johnson (1990), though such waves were permitted in their model.

In summary, in the first part of the Pedlosky and Spall paper, the steady upstream Rossby waves (Pedlosky and Spall 2015, their Figs. 6–9) are spurious because of inappropriate upstream and downstream boundary conditions and inappropriate representation of motion on the upstream side of the island. The solutions are interesting but are not the solutions to the problem as posed in their paper. In the second part, we are puzzled by the existence of the upstream waves in the authors’ time-dependent simulations, since they are also in contravention of linear Rossby wave theory.

REFERENCES


