A physically based, nondimensional parameter for discriminating between locations of freezing rain and ice pellets

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ABSTRACT

A nondimensional parameter is presented that can be used to help distinguish between conditions favorable for the occurrence of freezing rain and ice pellets. The parameter was derived from the well-established condition that most incidents of freezing rain and ice pellets are associated with a layer of above-freezing air elevated above a layer of below-freezing air adjacent to the earth’s surface and the requirement that any cloud ice must completely melt for freezing rain, otherwise ice pellets would result. The parameter was obtained from the ratio of the time available for melting to the time required for complete melting. The parameter was tested on the mesoscale thermodynamic conditions that existed with the 1990 St. Valentine’s Day ice storm that affected much of the Midwest and on a number of other episodes of freezing rain and ice pellets in the Midwest. Testing showed excellent spatial agreement between diagnosed and observed locations for freezing rain and ice pellets. An isonomogram is presented to allow the parameter to be easily used as a tool in determining winter precipitation type.

1. Introduction

Distinguishing between regions that will receive freezing rain from those that will receive ice pellets¹ presents a difficult challenge in short-range winter weather forecasting. This challenge exists even though the physical processes by which most instances of freezing rain and ice pellets occur have been recognized for a long period of time (e.g., Brooks 1920). Penn (1957) pointed out in a U.S. Weather Bureau forecasting guide that freezing rain and ice pellets are accompanied by a shallow wedge of subfreezing² air positioned adjacent to the earth’s surface that underlies an elevated layer of above-freezing air and that the extent to which ice particles melt in the elevated warm layer would determine whether freezing rain or ice pellets would reach the earth’s surface.

If an ice particle completely melts during its fall through the midlevel warm layer of cloud, raindrops fall out of cloud base into air that is subfreezing where, owing to a lack of ice nuclei (Fletcher 1962; Bigg 1953, 1955; Vali 1994), they supercool and reach the earth as freezing rain. If the ice particles only partially melt during their fall through the elevated warm layer, the presence of the ice in the liquid–ice hydrometeor al-

¹ By freezing rain, we mean rain that falls as supercooled raindrops that freeze on impact with the ground or other objects exposed at the earth’s surface. By ice pellets, we use the definition given in the Glossary of Meteorology (Huschke 1959), which states, “a type of precipitation consisting of transparent or translucent pellets of ice, 5 mm or less in diameter.”

² In this paper, air temperatures colder than or equal to 0°C are referred to as “subfreezing” or “freezing” temperatures although strictly speaking 0°C is the temperature at which ice melts.

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lows freezing to proceed in the cold air beneath cloud base so that ice pellets reach the earth.

The importance of a melting zone in the production of freezing rain and ice pellets has been noted in numerous other studies that focused on winter precipitation type (e.g., Young 1978 or Stewart and King 1987) and was clearly evident in composite temperature and dewpoint profiles constructed by Bocchieri (1980) from 127 freezing rain radiosondes (RAOBs) for 48 stations. Bocchieri’s research also showed instances of freezing drizzle when the sounding was everywhere colder than 0°C. These have since been attributed to a coalescence process strictly involving supercooled drops (Huffman and Norman 1988).

Model calculations by Stewart and McFarquhar (1987) of ice particles melting in an elevated warm layer clearly showed a progression of precipitation types from freezing rain, to ice pellets, to snow as melting diminished the warm layer. The presence of an elevated warm layer and degree of ice particle melting are reasons why thickness, the depth of the warm layer, and height of the freezing level above surface have provided useful guidance in distinguishing between where freezing rain and ice pellets will occur (Booth 1973; Boyden 1964; Burnash and Hug 1970; Glahn and Bocchieri 1975; Koolwine 1975; Lumb 1961, 1963; Mahaffy 1961; Murray 1952; Pandolfo 1957; Wagner 1957; Younkin 1967).

Bocchieri (1980) used the technique of regression estimation of events probability (Miller 1964) to relate RAOB parameters to diagnose winter precipitation type. Bocchieri’s set of linear regression equations had short-range predictive value to the extent that they could be used with either near-real-time or model output data. The probabilistic method did well at discriminating between rain and snow but did not perform so well at distinguishing between freezing rain and ice pellets.

This paper presents a physically based, nondimensional parameter that can be used as additional guidance to distinguish between where freezing rain and ice pellets will occur. Formulation of the parameter is presented followed by an application of it to a significant ice storm event. A qualitative evaluation based on a number of ice storm events is then discussed. An isonogram is introduced that allows the new methodology to be applied without having to resort to complex calculations. A few cautions in its use are enumerated. The paper concludes with a brief discussion of short- and moderate-range predictive values using observational and model data that are emerging with modernization of the U.S. National Weather Service.

2. Formulation

Figure 1 shows the conceptual model that was used to formulate the nondimensional parameter. Shown is a vertical cross section of a cloud whose top temperature is colder than 0°C. The figure is meant to illustrate how temperature varies in the vertical direction only with respect to cloud. The cloud straddles a midlevel warm layer, or melting zone, and temperatures beneath cloud base are colder than freezing. The critical factor that can decide between the production of freezing rain and ice pellets is whether an ice particle will completely melt in the elevated warm layer before entering the subfreezing air beneath cloud.

An indication of whether complete melting will occur can be obtained from the ratio, $\tau$, between the residence time, $t_{res}$, of a particle in the melting zone with a radius, $a$, and the time, $t_{pelt}$, required for complete melting. If the ice particle size distribution is characterized by a maximum particle radius, $a_0$ then the combination of layer depth, layer mean temperature, and resident time will define a value of $\tau \geq 1$, indicating that conditions are sufficient for complete melting of the largest and all smaller particles to favor freezing rain. Locations where the physical conditions of the warm layer define $\tau < 1$ suggest only partial melting of the largest particles. Hence, for $\tau < 1$ the extent to which ice pellets reaches the earth depends on the concentration of particles smaller than $a_0$ and the likelihood that smaller drops will survive evaporation and collection after falling out of the cloud.

The resident time of a particle in the melting zone can be estimated from the ratio of the vertical distance the particles must fall (i.e., depth of the warm layer, $\Delta Z_w$) and the velocity at which they fall through the warm layer:

$$t_{res} = \frac{\Delta Z_w}{U(a) - \bar{V}}, \quad (1)$$

![Fig. 1. Schematic diagram showing conceptual relationship between the elevated warm layer (melting layer), the lower layer of below 0°C air, and the position of vertical cloud cross section.](Unauthenticated | Downloaded 07/19/24 08:17 PM UTC)
where \( U(a) \) is particle terminal velocity and \( \bar{V} \) represents the mean vertical air motion. For simplicity, \( \bar{V} = 0 \) has been assumed although melting and particle loading would dictate otherwise (e.g., Szeto et al. 1988). Also assumed is that \( a \) and \( U(a) \) remain constant as a particle falls and melts, and that it does not change size by collection and/or condensation–evaporation mechanisms. By neglecting these aspects, Eq. (1) would be too low an estimate if the vertical air motion was upward and particle sizes decreased by evaporation and would be too high if the particle increased size and/or the vertical air motion was downward.

The time required for a particle to completely melt can be estimated from a balance between the rate of energy required to transform the solid to liquid and the energy required to transfer through the water layer and disposed of in the environment can be written as

\[
4\pi \rho_i L_i a^2 \frac{dr}{dt} = \frac{4\pi \alpha r k_T}{\alpha - r}(T_o - T_o(r)) \tag{2}
\]

where \( \rho_i \) is the density of ice, \( L_i \) the latent heat of melting, \( k_T \) is the thermal conductivity of water, \( T_o \) is the temperature at the ice/water interface assumed to be 273\(^\circ\)K, and \( T_o(r) \) is the temperature at the particle/environment interface, which is a function of the size of the ice core. Integration of Eq. (2) results in

\[
t_{\text{melt}} = \frac{L_i \rho_i}{k_T \alpha} \int_a^r \frac{r(a - r)dr}{T_o - T_o(r)} \tag{3}
\]

The temperature at the particle/environment interface, \( T_o(r) \) in Eq. (3), can be found iteratively from

\[
\frac{4\pi k_T a r}{(a - r)}[T_o - T_o(r)] = -4\pi \alpha k_T [T_o - T_o(r)] f_h - 4\pi a L_i D_v (\rho_{v,\infty} - \rho_{v,w}) \bar{f}_v , \tag{4}
\]

where \( k_T \) is the thermal conductivity of air, \( f_h \) and \( \bar{f}_v \) are ventilation coefficients for heat and vapor, \( D_v \) the diffusivity of vapor, \( L_a \) the latent heat of vaporization, and \( \rho_{v,\infty} - \rho_{v,w} \) the difference between the vapor density of the environment and the saturation vapor density at the particle’s surface.

The ratio of Eqs. (1) and (3) results in the nondimensional discriminator parameter

\[
\tau = \frac{t_{\text{res}}}{t_{\text{melt}}} = \frac{\Delta Z_k a}{(U(a) - \bar{V})L_i \rho_i \int_a^r \frac{r(a - r)dr}{T_o - T_o(r)}} \tag{5}
\]

Particle fall velocity, \( U(a) \), can be estimated from

\[
U(a) = c_1 - c_2 \exp(-c_3 a), \tag{6}
\]

where \( c_1 = 965 \text{ cm s}^{-1} \), \( c_2 = 1030 \text{ cm s}^{-1} \), and \( c_3 = 12 \text{ cm}^{-1} \) (Best 1950; Atlas et al. 1973). An empirical relationship for liquid raindrops has been used since this should better approximate the speed of a partially melted particle in free fall.

3. An example ZR-IP diagnosis

A freezing rain–ice pellets (ZR–IP) diagnosis is presented using the 1990 St. Valentine’s Day ice storm as an example. This ice storm was selected because certain aspects of it were poorly forecast and because it has received a great deal of research attention having occurred during four independent atmospheric field projects based from the Rocky Mountains to the eastern Great Lakes (Martner et al. 1992; Rasmussen et al. 1995; Rauber et al. 1994). A variety of severe weather was produced in addition to freezing rain and ice pellets, including heavy snowfall, thunderstorms, hail, and tornadoes. Thirty-five states were affected, resulting in an estimated $120 million in property damage.

Surface weather conditions were characterized by a cold front that passed through east-central Illinois (see Fig. 2) at 1200 UTC 14 February 1990. After passage, the front assumed an orientation parallel to the southwesterly upper-level 500-mb flow, resulting in a stationary overrunning precipitation pattern. The accumulation of ice was estimated to be between 0.15 and 0.65 cm, and occurred from three rain episodes between 2200 UTC 14 February and 1300 UTC 15 February, 1990. The total amount of precipitation measured at the cooperative station in Champaign, Illinois, was 3.3 cm (melted equivalent). Air temperatures at shelter level were only a few tenths of a degree colder than 0\(^\circ\)C during the entire course of the event: beginning at about –1\(^\circ\)C at 2300 UTC 14 February 1990 and steadily increasing to a few tenths of a degree colder than 0\(^\circ\)C when the rain ended at about 1200 UTC 15 February 1990.

A National Center for Atmospheric Research/Cross-Chain Loran Atmospheric Sounding System (NCAR/CLASS) sounding taken during the evening of 14 Feb-
ruary 1990 as part of the field operations of the University of Illinois Winter Precipitation Program is shown in Fig. 3 (Ramamurthy et al. 1991). The sounding shows the well-recognized cold-air layer near ground level at 990–925 mb with an elevated warm air layer above at 925–710 mb. In the cold-air layer temperatures reached a minimum of about −4°C (at ~930 mb), while temperatures aloft reached a maximum of +8°C (at ~840 mb). Cloud bases were estimated to be approximately 1 km (940 mb), and cloud tops were as high as 9 km.

In practice, a value for critical particle radius, \( a_c \), needs to be determined at each location in space where \( \tau \) is calculated. To gain a rough indication of what the critical ice particle radius may have been, we followed a procedure that used reflectivity data from the National Weather Service (NWS) radar summary along with established relationships between reflectivity and characteristics of the particle size distribution. The reflectivity relationship used was

\[
Z \text{ (dBZ)} = f(\lambda, N_0, a_{\text{min}}, a_{\text{max}}),
\]

where \( \lambda \) is the slope, \( N_0 \) is the intercept, and \( a_{\text{min}} \) and \( a_{\text{max}} \) are the minimum and maximum particle radii, respectively, of the size distribution. In this case \( a_{\text{max}} \) is equivalent to \( a_c \). A specific form of Eq. (7) has been found by Czys and Tang (1995), who integrated the defining equations for \( Z \) (e.g., Pruppacher and Klett 1978) over the finite drop size range \( a_{\text{min}} \) to \( a_{\text{max}} \). The integration was accomplished by assuming an exponential particle size distribution defined by

\[
N(a) = N_0 e^{-\lambda a},
\]

where \( N_0 \) is the intercept, \( \lambda \) is the slope, and \( a \) is the particle radius (Marshall and Palmer 1948). For the present problem, a Marshall–Palmer value of \( N_0 = 0.08 \text{ cm}^{-4} \) was used. Although observation has not always supported use of Marshall–Palmer values, especially for mixed phase regions and/or the condition of significant ice particle melting, in the absence of a better choice doing so allows the next step of the calculation to be taken.

The slope of the particle size distribution was obtained from the rainfall-rate relationship

\[
\lambda = 41 \mathcal{R}^{-0.21}
\]

given by Gunn and Marshall (1958), where \( \mathcal{R} \) is the rainfall rate (mm h\(^{-1}\)). Observed rainfall rates from rain gauge data were used with Eq. (9). Now that we have values for \( N_0 \) and \( \lambda \), and if we assume that \( a_{\text{min}} = 0 \), \( a_c \) can be computed directly from Eq. (7), which resulted in a critical radius of 400 \( \mu \text{m} \).

Values of \( \tau \) were calculated from Eq. (5) using 0000 UTC 15 February 1990 radiosonde data and are plotted in Fig. 4. The data for each sounding were integrated using a simple computer program that first determined
if a melting layer existed and, if it did, computed melting layer depth and mean temperature [where the mean temperature of the melting layer was also used for $T_0$ in Eq. (4)]. When no melting layer existed, $\Delta Z_m$ was set to 0 effectively setting $\tau$ to 0. The field of $\tau$ shown in Fig. 4 was then used with a surface isotherm analysis for the same time (see Fig. 5) to arrive at a final diagnosis. Surface temperature was used to rule out the possibility of making a diagnosis of freezing rain where surface temperatures warmer than 0°C would simply result in rain.

The final diagnosis is shown in Fig. 6. In general, the diagnosis indicates two elongated regions extending southwest to northeast with the northernmost region diagnosed to have ice pellets and the southernmost region to have freezing rain. The southern boundary of the region of freezing rain exactly corresponds to the location of the 0°C isotherm shown in Fig. 5. The northern boundary of the region for ice pellets corresponds
to the northern $\tau = 0$ line in Fig. 4. The boundary separating the regions of freezing rain and ice pellets corresponds to the $\tau = 1$ line in the region with surface temperatures colder than 0°C.

### 4. Preliminary evaluation

The observed distribution of precipitation type at 0000 UTC 15 February 1990 is shown in Fig. 7. The diagnosed and observed regions for freezing rain and ice pellets are in remarkably good agreement considering that a number of marginal assumptions were made, and in spite of the coarse spatial resolution of the radiosonde data.

The diagnosed field shows an area of freezing rain that almost exactly coincides with that observed except for a large region of false alarm in central Oklahoma and south-central Kansas. This region of false alarm occurred because it was an area without precipitation, and no attempt was made in the diagnosis to filter according to the presence or absence of precipitation. The diagnosis of ice pellets coincides with the region of observed except that it is not as narrow. This difference probably occurred because of the coarse spatial resolution of the sounding data. When conditions for freezing rain and ice pellets were diagnosed for the observation times of 1200 UTC 14 February 1990 and 1200 UTC 15 February 1990, corresponding to the beginning and end of the St. Valentine’s event, similar good agreement was found and the same types of qualitative errors were uncovered.

The methodology was further evaluated in a cursory forecast experiment conducted during the winter of 1995/96 with the cooperation of the National Weather Service Forecast Offices (NWSFOs) at St. Louis, Missouri, and Indianapolis, Indiana. At total of 17 freezing rain and ice pellets episodes were identified within the geographic domain represented in, for example, Fig. 7. As was the case for the St. Valentine’s ice storm, qualitatively the methodology tended to accurately identify areas where freezing rain and ice pellets did occur. Also in keeping with the St. Valentine’s example, false alarms resulted by not taking into consideration the areal distribution of precipitation. Another source of false alarm uncovered in the experiment was occasions when an elevated warm layer developed as a result of strong nocturnal cooling in the boundary layer in the absence of precipitation. A tendency for the diagnosis to have a less detailed spatial fit than observed was also noted in the experiment.

The preliminary evaluation leads to consideration of a few improvements. First, the simplified theoretical treatment for the heat balance equation governing melting given in Eq. (2) could better specify the effect that the ice—liquid mixture has on consumption of energy from the environment. Future research needs to be conducted so that the type of ice entering the top of the melting layer can be better taken into account. The present theory assumes that the ice is initially spherical when in reality it may start out as aggregates, or possibly large individual crystals that may or may not exhibit some degree of riming. It can be suspected that an aggregate would take longer to melt than another form of ice because of the delay induced by the time required for melting to collapse the aggregate into a spherical liquid with an ice core, as assumed in development of Eq. (2).

**GOES-8** satellite data could be used to improve the diagnosis by helping to delineate between the presence and absence of clouds and to estimate cloud-top temperature to determine the extent to which ice may be involved in the formation and evolution of precipitation. Further improvement can be made if values of the critical radius were determined over a higher spatial resolution and regular grid spacing using, for example, composite WSR-88D reflectivity data with rain-rate data. This would require future research to determine the best location within the echo structure to extract reflectivity for use and development of specific rain-rate relationships for freezing rain and ice pellets.

### 5. Application

Figure 8 was created to allow Eq. (5) to be applied simply. It shows isopleths of $\tau = 1$ computed over a range of critical ice particle radii, warm layer depths, and mean warm layer temperatures. In the 17 cases identified in the winter 1995/96 forecast experiment, depths and mean temperatures as large as about 5000 m and 10°C, respectively, were found. The $\tau = 1$ iso-

![Fig. 8. Isonomogram of $\tau = 1$ for different critical ice particle radii computed over a range of warm layer depths and mean temperatures.](image-url)
pleth for \( c_i = 400 \ \mu m \) has been highlighted because this radius proved to be adequate for the variety of freezing rain and ice pellets situations encountered in the forecast experiment.

A diagnosis can be made from Fig. 8 by first determining values for warm layer depth and mean layer temperature from radiosonde or another source of sounding data. For example, suppose that a warm layer depth of 2500 m and a mean layer temperature of +4°C were indicated. This combination indicates freezing rain because it intersects well above the \( T = 1 \) line for \( c_i = 400 \ \mu m \). Next, a judgment has to be made about the adequacy of assuming a value of 400 \( \mu m \) for the critical ice particle radius. This judgment could be based on a quantitative assessment using rain-rate and reflectivity data as was illustrated in section 3, or it could be qualitative based on inspection of WSR-88D reflectivity data. As one cautionary note, those using this radius must remember that it will fail to identify instances of freezing drizzle when the sounding is everywhere colder than 0°C. However, our experience from the 1995/96 forecast experiment suggests that this is a highly localized phenomena.

Figure 8 also gives a semblance of the sensitivity of Eq. (5) to the selection of critical radius. It clearly shows that a modest variation in critical radius will result in selecting one form of precipitation type over the other. Thus, it is interesting that a value of 400 \( \mu m \) seemed to apply so well to the 17 cases examined during the 1995/96 forecast experiment. This suggests the hypothesis that the natural variability of particle size distribution and the bright band associated with freezing rain and ice pellets is not as large as the sensitivity shown in Fig. 8. This accentuates the need for future additional microphysical observations through the bright band associated with freezing rain and ice pellets, to compare these observations with the bright band of other mesoscale structures, and to find a means to readily estimate critical ice particle radius using remote observations.

6. Conclusions

A nondimensional parameter was presented that can be used to assist in the diagnosis of conditions favorable for the occurrence of freezing rain and ice pellets. The parameter was derived from consideration of the time required for ice particles to melt in an elevated layer of above-freezing air to an estimate of the residence time of the ice particles in the warm layer. The parameter was applied to mesoscale thermodynamic conditions that existed with the 1990 St. Valentine's Day ice storm that affected much of the Midwest. This application showed agreement between diagnosed and observed locations for freezing rain and ice pellets. Agreement was also found in a preliminary evaluation based on 17 freezing rain and ice pellets events that occurred in the Midwest during the winter of 1995/96. The parameter can be applied to make short-range predictions using model output, such as that from the rapid update cycle model.

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