An Algorithm for Forecasting Mountain Wave–Related Turbulence in the Stratosphere

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ABSTRACT

A global mountain wave parameterization for prediction of wave-related displacements and turbulence is described. The parameterization is used with input from National Meteorological Center analyses of wind and temperature to examine small-scale disturbances encountered by the National Aeronautics and Space Administration high-altitude ER-2 during the Second Airborne Arctic Stratosphere Experiment. The magnitude and location of observed large wave events are well reproduced. A strong correlation is suggested between patches of moderate turbulence encountered by the ER-2 and locations where breaking mountain waves are predicted by the parameterization. These facts suggest that useful forecasts of global mountain wave activity, including wave-related clear-air turbulence, can be made quickly and inexpensively using our mountain wave parameterization with input from current numerical forecast models.

1. Introduction

The connection of stratospheric turbulence to mountain wave activity has long been recognized (cf. Eehnberger 1987). However, the complexities of mountain flows have prevented the development of useful forecasting tools for use on a global scale. Part of the difficulty lies in the complicated nature of topographic forcing. Real topography possesses a rich spatial spectrum that, like the spectrums of many natural phenomena, obeys a power law (Mark and Aronsen 1984). However, the topographic spectrum is not in general isotropic (Steyn and Ayotte 1985). In other words, there are directional differences (anisotropy) in the magnitude and scale of topographic variance. This anisotropy is clearly evident in the fact that much of earth’s topography is organized into relatively long, narrow ridges. The orientation of these ridges is an important variable in determining the wave response of the atmosphere to topography.

This study will describe improvements to a mountain wave parameterization scheme introduced in Bacmeister (1993) that allow reliable forecasts of mountain wave amplitudes in the upper troposphere and lower stratosphere to be made based on standard meteorological products from operational forecasting models such as the National Meteorological Center (NMC) T126 medium-range forecast (MRF) model. The basis for these mountain wave forecasts is a global, computer-generated list of prominent topographic ridges containing parameters such as latitude, longitude, orientation, approximate altitude, and approximate width for each ridge. The algorithm used to construct this dataset is described in section 2a of this study. Mountain wave quantities are calculated using the Wentzel–Kramers–Brillouin approximation with wave amplitudes limited to saturation values as done in earlier parameterizations (e.g., Macfarlane 1987). This calculation is described in detail in section 2b. In section 3 predicted wave amplitudes and turbulence intensities are compared with observations of wind and temperature collected by the National Aeronautics and Space Administration (NASA) ER-2’s during the second Airborne Arctic Stratosphere Experiment (AASE-II).

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2. Description of mountain wave parameterization

The theoretical basis for the parameterization used in this study is the same as that in Bacmeister (1993). However, significant improvements have been made to the description of topographic forcing used in the present parameterization. The study of Bacmeister (1993) examined the effects of topographic anisotropy (rigidness) on estimates of mountain wave drag in the middle atmosphere. The topographic forcing in this study was based on a box-by-box analysis of topographic features with scales of between 50 and 100 km in a grid box of size 250 km$^2 \times 250$ km$^2$. Only one ridge was assumed to exist within each grid box, and no attempt was made to segregate features by width or to identify features with scales smaller than 50 km. While this approach may have been adequate for an initial assessment of anisotropic effects and comparison with other coarse-grained gravity wave parameterizations, a more detailed model was needed for operational forecasting of flight conditions in the stratosphere and for subsequent comparison with airplane observations.

a. "Ridge-finding" algorithm

Topographic data was obtained from a global dataset compiled by the National Center for Atmospheric Research containing mean elevations within $10' \times 10'$ rectangles. This elevation data was interpolated onto a $5' \times 5'$ grid and combined with a $5' \times 5'$ bathymetry dataset by the U.S. Navy. The interpolated $5' \times 5'$ elevation dataset was used to produce a global list of ridges containing the location, orientation, and approximate height and width of each ridge. The analysis of the topographic data consisted of eight basic steps:

1) interpolation to grid with equal spacing (in km) in zonal and meridional directions
2) bandpass filtering
3) normalization by local mean variance
4) elimination of variance below chosen threshold value
5) comparison at each point with idealized ridglets at 36 different orientations
6) selection of "good" matches
7) estimate of actual terrain elevation
8) reconstruction of topography

Each step is described in detail in the following.

Step 1. Interpolation is performed to remove geometric distortions in an equally spaced latitude-longitude grid arising from the earth’s sphericity. This is accomplished in a crude way by interpolating the raw $5' \times 5'$ topographic data in $10'$ latitude strips onto a grid with equal spacing in kilometers in both directions at the strip center. Thus, in the case of a $10'$ strip centered at $60'$ the raw $4320 \times 120$ data array is interpolated onto a $2160 \times 120$ array before further processing. In general, the raw $4320 \times 120$ strips at latitude $\theta$ are interpolated to $INT(sin\theta/4320)$ 120 arrays, where $INT(z)$ refers to the largest integer smaller than $z$. The strip centers are spaced $5'$ apart in latitude.

Step 2. Bandpass filtering is accomplished by subtracting the results of two different square boxcar filters applied to the interpolated topographic data. This step controls the scale of feature identified by the subsequent steps in the algorithm. The set of ridges used in this study was determined from four different bandpassed filtered topographic datasets. The high-pass, $L_{hp}$, and low-pass, $L_{lp}$ limits for each of these are listed in Table 1. The result of this operation will be referred to as the topographic deviation and denoted $Dev_{topo}$.

Step 3. Next, a running mean of the absolute value of the bandpass-filtered topography is determined. This is done with a square boxcar filter of size $10L_{hp}$. The bandpass-filtered topography, $Dev_{topo}$, from step 2 is then divided by this quantity at each point to give a "normalized" topographic deviation at each point.

Step 4. The normalized topographic deviation from step 3 is examined at each point. Values greater than or equal to a threshold value of $+1.25$ are replaced with $1.00$, and values less than this threshold are replaced with $0$. The result of this operation will be referred to as the "skeleton" topography and denoted $Skel_{topo}$. The exact value of the threshold does not appear to have a major impact on the rest of the processing as long as it is between $1.00$ and $2.00$. If a much higher value is chosen, the algorithm becomes too discriminating, choosing only the highest isolated peaks in any region. If a threshold value lower than $1.00$ is chosen, the separation between major and minor terrain features is lost. Subsequent processing is more time consuming and appears to be less accurate since spurious "ridges" may be introduced connecting major and minor features. The selection of the "best" threshold was accomplished by trial and error.

Step 5. The skeleton topography, $Skel_{topo}$, is then compared with 36 idealized "ridglets" $R(x_i, y_j, x_0, y_0, n; L_{hp})_{n=1, \ldots, 36}$ at each point $(x_0, y_0)$ in the $Skel_{topo}$ array. The ridglet functions $R$ are defined to have a value of 1.00 within an $L_{hp} \times 3L_{hp}$ rectangle about a central point $(x_0, y_0)$ and a value of 0 for all $(x_i, y_i)$ outside of the rectangle. Thirty-six different trial orientations differing by $5'$ are used at each point. The comparison of the idealized ridglets with $Skel_{topo}$ at $(x_0, y_0)$ is accomplished by calculating the sum.

<table>
<thead>
<tr>
<th>Step 2 of ridge-finding algorithm.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High-pass limit ($L_{hp}$)</strong></td>
<td><strong>Low-pass limit ($L_{lp}$)</strong></td>
</tr>
<tr>
<td>80 km</td>
<td>185 km</td>
</tr>
<tr>
<td>55 km</td>
<td>128 km</td>
</tr>
<tr>
<td>40 km</td>
<td>98 km</td>
</tr>
<tr>
<td>28 km</td>
<td>58 km</td>
</tr>
</tbody>
</table>
\[ f(x_0, y_0, n) = \frac{\sum R'(x_i, y_j, x_0, y_0, n_0) \text{Skele}_{\text{topo}}(x_i, y_j)}{\left( \sum [R'(x_i, y_j, x_0, y_0, n_0)]^2 \right)^{1/2}} \]  

(1)

for each orientation \( n \), where \((\quad)'\) refers to the deviation of the quantity from a running mean over a \(3L_{\text{hp}} \times 3L_{\text{hp}} \) square.

Step 6. The sums \( f(x_0, y_0, n) \) in step 5 can vary from -1 to +1. Unfortunately, the ridge functions \( R \) are not orthogonal. However, the value of the sum \( f \) increases as the similarity between the underlying topography and the trial ridge function increases. Thus, in order to determine the orientation of the underlying terrain features at each point \((x_0, y_0)\), the 36 sums \( f(x_0, y_0, n) \) are searched for the orientation index \( n_0 \) associated with the highest value of \( f \). Next, all points \((x_0, y_0)\) in the topographic array are searched for negative values of \( f(x_0, y_0, n_0) \). These negative values are discarded. This eliminates “trenches” or canyons from consideration. Finally, each \( L_{\text{hp}} \times L_{\text{hp}} \) square in the topography array is examined separately for the highest value of \( f \) contained within, and only this value is kept. This is done to eliminate as much redundancy as possible in the list of ridges before further use.

Step 7. Once a list of the location and orientation of possible ridges is obtained, an estimate of actual elevation changes across each ridge is made. The estimate is obtained from the sum

\[ a(x_0, y_0, n_0) = \frac{\sum R'(x_i, y_j, x_0, y_0, n_0) \text{Dev}_{\text{topo}}(x_i, y_j)}{\left( \sum [R'(x_i, y_j, x_0, y_0, n_0)]^2 \right)^{1/2}} \]  

(2)

The quantity \( a \) is simply the projection of the topographic deviation from step 2 onto the ridge function \( R'(x_i, y_j, x_0, y_0, n_0) \). The relationship of \( a \) to the actual valley-to-peak ridge height depends on details of the terrain shape. However, for a wide variety of real and idealized ridge profiles, \( a \) is between 1/3 and 1/4 of the actual ridge height.

Step 8. At this point in the processing each choice of bandpass parameters \( L_{\text{hp}} \) and \( L_{\text{dp}} \) is associated with a list of topographic features. Each feature or ridgelet is assigned a position \((x_k, y_k)\), an orientation \( n_k \), a “ridginess” index \( f(x_k, y_k, n_k) \), and a ridge-height parameter \( a(x_k, y_k, n_k) \) where the subscript \( k \) denotes the \( k \)th ridge in the list. The full list of ridges contains redundancy since the functions \( R \) at different points may overlap. The basic strategy for dealing with redundancy in the list of ridges is as follows. First, the list is ordered by the size of the height parameter \( a \) with the highest ridge coming first. Next, the ridges are laid one-by-one, starting with the highest, onto a flat surface at the appropriate location \((x_k, y_k)\). Each ridgelet is represented simply by the expression

\[ \text{ridgelet}_k(x_i, y_j) = 4a(x_k, y_k, n_k)R(x_i, y_j, x_k, y_k, n_k; L_{\text{hp}, k}). \]  

(3)

If some of the area covered by the latest ridge is already covered by previous ridges, the newest ridge does not add elevation to that area. A “weight” is determined from the fraction of the new ridge that is not covered by previous, higher ridges. If 25% of the new ridge is exposed and 75% is covered by higher ridges, then the new ridge is assigned a weight of 0.25, and so on. This is repeated until all ridgelets in the list have been placed on the surface. At the end of this procedure, redundancies within a given list of ridges have been accounted for by the fact that in any pair of overlapping ridges at least one of the ridges will have weight less than 1.0.

Clearly, redundancies within lists of ridgelets of the same width should be accounted for. However, the situation across lists corresponding to different ridge widths is not clear. A single point in the topographic array may be covered by several ridgelets of different widths. In the following study we have assumed that such redundancy reflects true topographic structure and may thus be ignored.

A straightforward approach involving fast Fourier transforms (FFTs) of topographic elevation data was not used to construct the ridge dataset for several reasons. First, a large number of modes is required to obtain the same accuracy in the estimate of dominant ridge orientation. Thus, either a large subarray of topography must be used or small subarrays of topography must be interpolated onto a finer grid. Using a large subdomain both increases the possibility of multiple ridges in a given subarray and reduces the accuracy with which individual features can be localized in physical space. If small subarrays of topographic data are used, artificial edges at the subarray boundaries must be dealt with in some fashion. Second, in either case, FFTs cannot accurately represent the orientation features with relatively small aspect ratios such as 3:1. However, such features may still exhibit “ridgelike” behavior in a stratified flow (Hines 1988).

The ridge-finding algorithm used in this study is in no sense a standard procedure. In practice the algorithm was interactively tuned by applying it to several mountainous regions and adjusting parameters until satisfactory results were obtained. It should be emphasized that this tuning was done only at the beginning and that a single set of parameters was used to obtain the global list of ridges. Results from several steps in the ridge-finding algorithm applied to the topography of Alaska and northwestern Canada are shown in Fig. 1. Figures 1a–c show results for bandpass parameters \( L_{\text{hp}} = 55 \) km and \( L_{\text{dp}} = 128 \) km. The final result for this region including features from all four width ranges is shown in Fig. 1d. Major topographic features such as the Alaska Range and the Canadian Rockies are clearly evident and accurately reproduced in the final reconstruction. Secondary features such as topography on the Aleutian Islands and Queen Charlotte Islands (132°W, 53°N) are also captured. In section 3 it will
be shown that such "secondary" topography, in this case the Laurentian Mountains and Notre Dame Mountains of southern Quebec, may produce noticeable turbulence at stratospheric altitudes.

b. Estimate of mountain wave activity

The list of topographic ridges developed in section 2a allows global mountain wave activity to be estimated on a ridge-by-ridge basis. Background wind and stratification near each ridge are determined from NMC T126 MRF forecast and/or NMC Climate Analysis Center analysis fields. In the calculations discussed here we simply use data from the NMC grid point nearest to each ridge center \((x_k, y_k)\). The estimate of wave amplitudes is made essentially as in Bacmeister (1993). First, the profile of the wind component perpendicular to the \(k\)th ridge \(U_{\perp;k}(z)\) is calculated using the ridge orientation \(n_k\) found in section 2a. The profile of stratification frequency or buoyancy frequency above each ridge, \(\nu_k(z)\), is estimated from the local NMC potential temperature profile \(\Theta_k(z) = (g \delta \Theta_k/\Theta_k)^{1/2}\). We assume that the waves launched by each ridge are in steady state and are purely two-dimensional with wave crests parallel to the generating ridge at all levels. We also assume the waves are hydrostatic so that wave activity is generally localized over the forcing topography. The average momentum flux profile over any ridge can be approximated in terms of the wave vertical displacement profile

\[
\phi_k(z) \sim \alpha \rho(z) N_k(z) U_{\perp;k}(z) \frac{\delta_k(z)^2}{L},
\]

where \(\delta_k(z)\) is the profile of wave-induced vertical displacement above the \(k\)th ridge and \(\rho(z)\) is the background atmospheric density profile, which we assume is simply proportional to pressure. The parameter \(\alpha\) is a dimensionless factor that depends on ridge shape, and \(L\) is horizontal length representing the extent of the wave disturbance. As will be seen below, these last two parameters drop out in the calculation of the wave displacement profile.

We assume further that wave momentum flux remains constant with height until wave breaking occurs. This will be the case for hydrostatic waves in a slowly varying (with altitude) atmosphere. Our criterion for wave breaking is based on simulations of two-dimensional flow over topography, which suggest that wave amplitudes do not exceed the local saturation limit given by

\[
\delta_{\text{sat},k}(z) = U_{\perp;k}(z)/N_k(z).
\]

Using (4) and (5), we can construct an approximate wave displacement profile as follows. Given \(\delta_k(z)\) at some analysis level, we use (4) to make a provisional guess of the displacement at the next level \(\delta_k^*(z + \Delta z)\). Then, if the provisional \(\delta_k^*(z + \Delta z)\) exceeds the saturation limit (5), it is arbitrarily reduced to the saturation limit before proceeding to the next level; that is,

\[
\delta_k^*(z + \Delta z) = \delta_k(z) \left[ \frac{\rho(z) N_k(z) U_{\perp;k}(z)}{\rho(z + \Delta z) N_k(z + \Delta z) U_{\perp;k}(z + \Delta z)} \right]^{1/2}
\]

\[
\delta_k(z + \Delta z) = \text{MIN} [\delta_k^*(z + \Delta z), \delta_{\text{sat},k}(z + \Delta z)].
\]

To complete the determination of \(\delta_k(z)\), the wave displacement at the surface must be known. For the lower boundary we use

\[
\delta_k(z_0) = \text{MIN} [4a_k, U_{\perp;k}(z_0)/N_k(z_0)],
\]
where \( a_k \) is the ridge-height parameter defined in (2). The lowest level \( z_0 \) used over a particular ridge depends on the estimated height of a particular ridge above sea level. The total height of ridge \( \phi(k) \) is estimated as \( H_k + 4a_k \), where \( H_k \) is the mean elevation in a 2.5° × 2.5° box containing the center of ridge \( \phi(k) \). We attempt to use wind and stratification near mountain-top level in each calculation; thus, the closest analysis/forecast level to mountain-top level is used as \( z_0 \). Possible refinements of this algorithm that include the possibility of turning by strong jets will be discussed in section 3.

Once profiles of wave-induced vertical displacements \( \delta_k(z) \) over each ridge are obtained, estimates of other wave quantities can be made. Wave-induced potential temperature perturbations are estimated according to

\[
\theta_k(z) = \delta_k(z) \Theta,
\]

where \( \Theta \) is the background potential temperature from the NMC analysis/forecast.

c. Estimate of wave-induced turbulence

The mechanisms that actually limit wave amplitudes to the saturation limit (5) are still controversial. However, it is likely that when wave amplitudes approach the saturation limit at least some of the wave energy will go into localized convective and shear instabilities. This assertion is well supported by both two-dimensional numerical simulations and linear wave theories. Thus, in the model described here, wave-induced turbulence is forecast whenever saturation is invoked to limit wave amplitudes in (6) and (7). Furthermore, we assume ad hoc that the intensity of the turbulence in a layer is proportional to the amount of momentum flux lost by the wave within that layer; that is,

\[
K_{E_{\text{Turbulence}}} (z + \Delta z/2) \propto \phi_k(z) - \phi_k(z + \Delta z),
\]

where \( \phi_k(z) \) is the momentum flux defined in (4) and \( K_{E_{\text{Turbulence}}} \) is a representative magnitude of the kinetic energy in small-scale (<1000 m) motions in a given layer.

At this stage, values must be assigned to the parameters \( \alpha \) and \( L \) in (4). We assume \( \alpha \approx 1.5 \) based on two-dimensional, numerical simulations as suggested in Pierrehumbert (1987). This value is based on experiments over smooth symmetric obstacles, so its general applicability is rightly questioned. However, quantitative estimates for \( \alpha \) in flow over realistic obstacles are not available. We assume that the length scale \( L \) is of secondary importance in estimating turbulence intensities in breaking waves. This is because the production of turbulence along a section of a breaking wave will depend on the total flux of wave momentum flux from below rather than on local momentum flux densities. The total momentum flux in a hydrostatic wave is independent of the obstacle width. For definiteness, we assign \( L \) a value of 50 km.

3. Model verification

Three clear examples of encounters with mountain waves in the stratosphere occurred during the AASE-II campaign. Two of these encounters were accompanied by pilot reports of moderate turbulence. For the purpose of model verification, global mountain wave predictions will be made below using NMC 18-level analyses of wind and temperature for each day on which a mountain wave was encountered by the ER-2. The predictions of the model along the ER-2 flight track will be compared with wind and potential temperature observations collected by on-board instruments during each of the wave encounters. The wind data used is from the Meteorological Measurement System (MMS), which provides three components of wind as well as temperature and pressure at a frequency of 1 Hz (Chan et al. 1989). Temperature and potential temperature profiles are derived from Microwave Temperature Profiler (MTP) data. The MTP provides a temperature profile composed of 15 independent measurements extending approximately 3 km above and below the aircraft (Gary 1989). A complete profile is obtained once every 9 s. This can be integrated vertically after suitable averaging to obtain pressure and potential temperature profiles. The individual potential temperature profiles can be joined along the flight track to obtain a cross section illustrating the vertical displacements of potential temperature surfaces along the flight path.

a. 14 October 1991 mountain wave observation

On 14 October 1991 the NASA ER-2 was ferried back to its home base at Moffet Field, California, from the AASE-II mission site at Fairbanks, Alaska. The plane took off from Fairbanks at about 65 000 s Universal Time (UT), or 1800 UTC, and flew north for 0.5 h before turning south. Figure 2a shows a portion of the ER-2 flight track over Alaska and western Canada with an overlayed map of turbulence potential at 70 mb for 14 October 1991. On this flight the ER-2 flew through regions where relatively high turbulence intensities were predicted. In fact, the pilot reported moderate turbulence and rapid temperature fluctuations over the coastal ranges of southern Alaska. These conditions were encountered between 68 000 s UT and 70 000 s UT. The position of the aircraft at 68 000 s UT and 70 000 s UT is indicated by the letters \( \text{A} \) and \( \text{B} \), respectively.

The turbulence potential map displayed in Fig. 2a consists of gray shaded symbols representing individual ridges from the global list described in section 2a. The darkness of each symbol denotes the possible intensity of wave-induced turbulence over each ridge, with the darkest symbols indicating the strongest possible turbulence. We use the assumption in (10) to predict the turbulence generated by breaking mountain waves at 70 mb, the standard pressure level nearest to ER-2.
Fig. 2. (a) Map of predicted turbulence potential $\Delta \phi_{MAX}$ at 70 mb and ER-2 flight track on 14 October 1991. ER-2 positions at 68 000 s UT and 70 000 s UT are denoted by A and B, respectively. Between these times the MMS and MTP instruments recorded large, rapid fluctuations in temperature and vertical wind. Dashed contours show 70-mb geopotential height in meters. (b) Map of wave-induced, isentropic displacements predicted by the parameterization for 14 October 1991. (c) Potential temperature as a function of time and height observed from the ER-2 on 14 October 1991. Potential temperature contours are drawn every 10 K. The light, dashed line superimposed on contours shows ER-2 altitude during the flight. The thin curve starting at 12 7.5 km shows predicted isentropic displacements at ER-2 altitude along the flight track. A bias of 22.5 km was added to displacements for display purposes. (d) The lower curve shows variance in 5-s vertical velocity fluctuations averaged over 100 s; that is, $\left\{ \left[ w(i) + 5 \text{ sec} - w(i) \right]^2 \right\}_{100 \text{ sec}}$ as function of time on 14 October 1991. Upper curve shows $\Delta \phi_{MAX}$ predicted along ER-2 flight track with a constant value of 2 added for display.

cruising altitude. The potential intensity of wave-induced turbulence near 70 mb is determined by calculating the differences:

$$\Delta \phi_\perp(70 \text{ mb}) = \phi(100 \text{ mb}) - \phi(70 \text{ mb})$$

and

$$\Delta \phi_\parallel(70 \text{ mb}) = \phi(70 \text{ mb}) - \phi(50 \text{ mb}),$$

where $\phi$ is the wave momentum flux in $(\text{m s}^{-1})^2 \text{ (kg m}^{-2})$ defined in (4). The larger of the two differences,

$$\Delta \phi_{MAX}(70 \text{ mb}) = \text{MAX} [\Delta \phi_\perp(70 \text{ mb}), \Delta \phi_\parallel(70 \text{ mb})],$$

is chosen to represent the potential for wave-induced turbulence. Thus, the darkness of each symbol in Fig. 2a indicates the largest predicted deposition of momentum flux by mountain waves in the two layers immediately above and below 70 mb. The size and orientation of each symbol shows the approximate size and orientation of the ridge element generating the wave disturbance. For hydrostatic waves generated by ridges, the wave energy is confined near the forcing
obstacle; thus, the map gives an approximate idea of where turbulence may be expected. Figure 2b shows a similar map for wave-induced, isentropic displacements predicted by the mountain wave parameterization.

Figure 2c shows the MTP potential temperature cross section between 16 and 24 km as well as an overlayed trace of aircraft altitude for the entire flight of 14 October 1991. The climb out of Fairbanks is evident about 67 000 s UT, as is the rapid descent into Moffet Field beginning at about 86 000 s UT. Between 73 000 s UT and 77 000 s UT the ER-2 performed a “science dive” in order to obtain vertical profiles of measured quantities. The bottom of the dive occurred at about 15 km. The ER-2 was allowed to cruise climb during the mission, so that by the time final descent into Moffet Field began the plane had reached an altitude of nearly 2 km. Similar flight plans were used for all of the ER-2 missions conducted during AASE-II large vertical isentropic displacements; approximately 500 km were recorded by the MTP between 68 000 s and 70 000 s UT as the plane crossed over the Alaska, Wrangell, and Chugach Ranges in southeastern Alaska. The displacements display a high degree of vertical coherence in the 6-km layer sampled by the MTP, which suggests that they form part of a deeper disturbance. The location of these displacements corresponds closely with the zone of high mountain wave activity predicted by the model (Figs. 2a,b).

To easily project results of the mountain wave model onto the ER-2 flight track we assume an elliptical influence function for each ridge. This function has a value of 1 along the axis of each ridge and falls off exponentially with total distance from the end points of the ridge. The relation of the influence function to the ridge symbols used in Figs. 2a,b is illustrated in Fig. 3. This function is multiplied times the peak values of isentropic displacement and turbulence potential predicted by the mountain wave parameterization to obtain a two-dimensional distribution of these quantities on each standard pressure surface. Then, the nearest standard pressure level to each point along the ER-2 flight track is found.

The vertical isentropic displacements predicted by the global mountain wave model along the ER-2 flight track are shown in Fig. 2c for comparison with the MTP cross section. The predicted isentropic displacements shown in Fig. 2c have been added to a constant value of 22.5 km for display purposes. It should be kept in mind that the parameterization predicts only peak wave amplitudes. No phase information is included. Thus, the sign of displacements is not predicted. In Fig. 2c all predicted isentropic displacements are arbitrarily assigned a positive sign. Nevertheless, Fig. 2c shows that there is rough agreement between predictions and observations in both the amplitude and location of large vertical displacements. The parameterization somewhat overpredicts the amplitude of the largest displacements. However, the observed displacements may depend sensitively on the exact position (to within 10 km) of the plane relative to each mountain wave. The current parameterization cannot resolve detailed structure within each wave. Therefore, exact agreement between predictions and observations cannot be expected.

To obtain an objective measure of the turbulence intensities encountered by the ER-2, rapid fluctuations in MMS vertical wind data were examined. Figure 2d shows variance in 5-s vertical velocity fluctuations averaged over a 100-s window; that is,

\[ \text{Var}_5(w) = \{ [w(t + 5 \text{ sec}) - w(t)]^2 \}_{100 \text{ sec}} \]

for the 14 October 1991 ferry flight. Variance far in excess of background levels was observed between 68 000 s and 70 000 s UT as the ER-2 flew through the mountain waves over southeast Alaska. Figure 2c also shows the predicted turbulence potential \( \Delta \phi_{\text{MAX}} \) at the standard pressure level closest to the aircraft altitude. For display purposes, the zero line for \( \Delta \phi_{\text{MAX}} \) in Fig. 2c has been moved to 2 on the \( y \) axis. The close agreement in peak amplitudes of wind variance in \((\text{m s}^{-1})^2\) and \( \Delta \phi_{\text{MAX}} \) is a fortuitous consequence of our choices for \( \alpha \) and \( L \) in (4) and of the air density near 70 mb. It should not be expected to hold at other pressure levels. However, the close agreement between the location of the predicted “turbulence” and the location of the large values in vertical wind variance is meaningful and suggests that the turbulence encountered on this flight is indeed the result of breaking, large-amplitude mountain waves.

**b. 6 January 1992 mountain wave observation**

On 6 January 1992 the ER-2 flew northeast from the AASE-II mission base at Bangor, Maine, across the southern tip of Greenland and back along the same track. Figure 4a shows the ER-2 flight track and turbulence potential map for 6 January 1992. No potential for turbulence is predicted along the ER-2 track, although maps of predicted wave-induced, isentropic displacements, shown in Fig. 4b, indicate 500–1000-m displacements over southern Greenland. Pilot reports for this flight list no turbulence or “chop.” However, altitude traces for this flight show a brief period of rapid but smooth ascent and descent over the tip of Greenland. This description is consistent with a large but nonturbulent mountain wave near the southern tip of Greenland as predicted by the parameterization. This wavelike feature was encountered on the outbound leg of the flight track at 58 000 s UT and again around 61 000 s UT on the inbound leg. The position of the aircraft at these times is indicated by the symbols “A” and “B” on the map.
The MTP cross section for this flight, in Fig. 4c, shows moderate wave activity between 57,000 s and 62,000 s UT as the ER-2 flew near Greenland. A large isentropic displacement, 800 m to 1 km, is evident at around 58,000 s UT and again at 61,000 s UT, as the aircraft flew over the southern tip of Greenland. The predicted peak, isentropic displacements for this day show large amplitudes, about 800 m, at the same locations. The amplitude of the predicted peak, isentropic displacements is roughly correct. However, the predicted displacements occupy a smaller portion of the flight track than was observed.

Figure 4d shows the 5-s, vertical wind variance for 6 January 1992. Despite the absence of reported turbulence on this flight, weak but significant enhancement of vertical wind variability occurs between 57,000 s and 62,000 s UT. The origin of this enhanced variability is unclear. It appears to coincide with “waviness” in the MTP section and may represent an enhancement in background variability over rough terrain (Nastrom and Fritts 1992) or an enhancement due to the nearness of the polar vortex edge (McIntyre 1989). In any case, we feel it is unlikely that the increase in small-scale variability near Greenland on this day was directly due to the breakdown of large-amplitude mountain waves. The lack of strong, wave-induced turbulence at flight level despite the presence of large wave displacements near Greenland may be a consequence of the strong, ridge-perpendicular wind (40 m s⁻¹ westerlies) present at flight level during this event. Using the expression for wave saturation amplitude in (5) and assuming a stratospheric $N \approx 2.0 \times 10^{-2}$ s⁻¹, this wind speed implies a saturation amplitude of nearly 2.0 km for mountain waves. Thus, both the observed and predicted isentropic displacements are less than 50% of the approximate saturation amplitude. This
suggests that wave breaking did not occur near flight level.

c. **18 March 1992 mountain wave observation**

On 18 March 1992 the ER-2 flew north from Bangor to Baffin Island and back. On the climb out of Bangor moderate turbulence was reported by the pilot as the plane flew near the St. Lawrence River in southeast Quebec. Figure 5a shows the southern portion of the ER-2 flight track and a turbulence potential map for 18 March 1992 at 100 mb. The location of the reported turbulence coincides approximately with the symbol A on the flight track. The map of turbulence potential shows relatively high values occurring on both sides of the St. Lawrence River, coinciding well with the location of the reported turbulence. The remaining symbols B and C refer to features that will be discussed below. Figure 5b shows a map of predicted isentropic displacements at 70 mb for 18 March 1992. Wave-induced isentropic displacements of about 500 m are predicted by the parameterization over southeast Quebec.

The MTP section for this flight, in Fig. 5c, shows evidence of a large vertical isentropic displacement approximately 500 m between 48 000 s and 49 000 s UT at an altitude of 17.5 km during the climb out of Bangor. Some degree of vertical coherence is evident in this feature. This wave appears to have been accurately predicted by the model both in location and amplitude. On the
descent back into Bangor there are again suggestions of coherent vertical displacements around 73 000 s UT as the plane crosses over point A again. However, the rapid descent of the plane makes these hard to distinguish in the MTP cross section. From the map in Fig. 5b it can be seen that the disturbance at A corresponds to ridges on the southeast side of the St. Lawrence River. These are the Notre Dame Mountains, a long, relatively low ridge with typical peak elevations of 650 m. From Figs. 5a,b it can be seen that the ER-2 passes directly across the ridge. So, a reasonably complete sampling of the wave disturbance should have occurred. We conclude that the disturbance at A is a mountain wave generated by the Notre Dame Mountains.

There is another weak but vertically coherent disturbance in the MTP cross section around 51 000 s UT and again around 71 000 s UT, which is denoted by C. Isentropic displacements of approximately 500 m are predicted at this location. The amplitude of the observed wave is difficult to determine but is clearly much less than the predicted value. The topographic features responsible for the overpredicted displacements at C appear to be 800–1000-m hills near Monts Otish (52°N, 70°W) in south-central Quebec. For isolated, narrow features such as these, the assumptions of high anisotropy rigidity and hydrostatic balance may both fail. Thus, the algorithm described in (4)–(8) may no longer be a valid approximate description of wave behavior. Both, nonhydrostatic effects and low rigidity will lead to horizontal dispersion of wave energy. This will tend to lower wave amplitudes immediately above the forcing topography to values below
those predicted by the model. On the other hand, there is strong evidence to suggest that the wave encountered at A was generated by the Notre Dame Mountains. This is a somewhat broader and longer topographic feature than the isolated hills near Monts Otish, so that the assumptions made in (4)–(8) about wave behavior are more closely satisfied.

A third wave with isentropic displacements of approximately 200 m is predicted at around 50 000 s and again at 72 000 s UT, at the times and locations denoted by B. However, no corresponding disturbance can be found in the MTP cross section. From the maps in Fig. 5a,b it appears that the predicted peaks at B are associated with a large wave predicted over the Laurentian Mountains on the northwest side of the St. Lawrence River. The low amplitude of this wave when projected onto the ER-2 flight track is a consequence of the influence function shown in Fig. 3 and the distance of the flight track from the ridge crest.

Figure 5d shows observed 5-s variability in vertical wind and predicted \( \Delta \phi_{\text{MAX}} \) for 18 March 1992. There is large \( \Delta \phi_{\text{MAX}} \) predicted near aircraft altitude between 48 000 s and 49 000 s UT and again around 73 000 s UT (denoted by A) as the ER-2 flew over the Notre Dame Mountains. These periods of high variability in vertical wind correspond closely to periods of high \( \Delta \phi_{\text{MAX}} \) predicted by the mountain wave parameterization. The strongest peak in the vertical wind variability also corresponds closely with the large isentropic displacement seen in the MTP section for this flight. This suggests that the turbulence encountered at these locations is due to the presence of a mountain wave forced by the Notre Dame Mountains, which is breaking in the 100–70-mb layer. As was the case for the 14 October 1991 flight, there is also rough agreement between the peak amplitudes of the variance in \((\text{m s}^{-1})^2\) and that of \(\Delta \phi\) in \((\text{m s}^{-1})^2\) (kg m m\(^{-3}\)). Although it is premature to draw conclusions from these examples, these cases suggest a coincidental but possibly useful proportionality between \(\Delta \phi\) and 5-s vertical wind variance close to the 70-mb level with a proportionality constant close to 1 (kg m m\(^{-3}\)).

Another large peak in \(\Delta \phi_{\text{MAX}}\) is predicted at 50 000 s UT (the first B), which is not clearly associated with a significant increase in vertical wind variability. This peak results from very large predicted momentum deposition of approximately 3 \((\text{m s}^{-1})^2\) (kg m m\(^{-3}\)) over the Laurentian Mountains between 100 and 70 mb. Interestingly, the predicted amplitude of this peak is much smaller for the return leg of the flight. This is simply a result of different aircraft altitudes at B during the outbound and inbound legs of the flight. On the inbound leg the ER-2 performed a cruise climb as can be seen in the altitude trace in Fig. 5c. By 72 000 s UT, the ER-2 had reached an altitude of about 20.5 km, which is close to the 50-mb surface at this location. Profiles of predicted wave momentum flux deposition over the Laurentians show large values below 70 mb but only small values above 70 mb. Thus, the deposition of wave momentum as flux is small in both the 70–50-mb layer and the 50–30-mb layer, and as a result, low turbulence potential is predicted.

Both the turbulence and isentropic displacements incorrectly predicted at B are associated with ridges in the Laurentian Mountains. In the case of the Laurentians, model assumptions of hydrostatic flow and ridgelike topography appear to be well satisfied. So an error would have to be ascribed to other physical processes left out of the model. However, it is also possible that the erroneously large values of turbulence potential and isentropic displacement calculated along the ER-2 flight track are due to problems in the influence function (Fig. 3) assumed to describe the horizontal distribution of variables predicted by the mountain wave parameterization. On the flight of 18 March 1992 the ER-2 did not fly directly over the Laurentians, the main ridge crest. This is apparent in Figs. 5a,b, which show that the ER-2 flight track did not actually cross the centerline of the ridge symbol corresponding to the Laurentians but stayed well to the east of it. Thus, the amount of wave activity calculated along the flight track will depend sensitively on the distribution of wave activity assumed to exist in the horizontal. Furthermore, measurements of a wave over the Laurentians on 18 March 1992 would have been prone to the same sensitivity to the exact position of the flight track relative to the wave. Thus, although the results of the mountain wave forecast in this case are not encouraging, we do not feel that they reflect a basic flaw in the technique.

The successfully predicted mountain wave encounter over the Notre Dame Mountains is noteworthy because it strongly suggests that even relatively minor topographic features may create significant turbulence in the lower stratosphere. This fact has implications for climate modeling as well as for turbulence forecasting. Parameterizations of mountain wave effects currently used in global forecasting and climate models use grid-box averages of topographic variance to estimate wave forcing. Even at the highest resolutions currently employed \(\sim 1° \times 1°\), this averaging may underestimate the potential of secondary features, such as the Notre Dame Mountains, to exert significant drag on the atmosphere.

4. Possible improvements to the model

The mountain wave parameterization described above relies on a simplified description of mountain wave behavior to predict wave amplitudes above individual topographic features. An ad hoc assumption is made concerning the production of turbulence by breaking mountain waves. To some degree such simplifications are unavoidable in constructing a global mountain wave parameterization using inputs from current numerical forecasting models. However, we feel improvements can be made in certain aspects of the parameterization—in particular, to the estimate of turbulence production by wave breaking.
a. Production of turbulence

The parameterization currently assumes a simple linear relationship between the momentum flux deposition in a layer, $\Delta \phi$, and turbulence intensity. This ignores possible interactions between the wave and the background wind. It is likely that waves breaking in a strongly sheared flow will produce stronger turbulence than those breaking in a uniform wind. Strong shears are generally thought to be rare in the stratosphere. However, it appears that planetary waves in the stratosphere can occasionally create relatively strong vertical gradients in on-ridge winds over individual ridges, as is illustrated in Fig. 6. Figure 6a shows NMC analyses of geopotential height for 16 January 1992 over eastern Asia at 100 and 50 mb. The geostrophic wind in this situation weakened and changed direction considerably between 100 and 50 mb due to the presence of what appears to be a synoptic wave with amplitude increasing as a function of height. Thus, on-ridge winds decreased sharply with altitude between 100 and 50 mb for ridges in Korea and Japan. This drop in on-ridge wind speeds is reflected in the high turbulence potential due to wave breaking predicted at 70 mb for these regions (Fig. 6b). Breaking mountain waves in this situation may have had enhanced levels of turbulence as a result of large shear in the background flow. Preliminary estimates of such an enhancement can be made with high-resolution, two-dimensional, numerical models such as those employed in studies of downslope windstorms (Peltier and Clark 1979; Durran 1986; Bacmeister and Pierrehumbert 1988).

b. Downstream advection of turbulence

Several incidences of light to moderate chop over ocean were reported by ER-2 pilots during AASE-II. Several of these occurred 100–500 km downwind of areas where strong turbulence due to breaking mountain waves was forecast by the parameterization. It is possible that advection of turbulent kinetic energy away from breaking waves is an important source of turbulence away from mountainous terrain. Estimates of decay rates for turbulence in stratified flow are available. These can be combined with turbulence production estimates from nonlinear simulations to give quantitative predictions for the downwind extent and intensity of turbulent layers produced by breaking mountain waves.

c. Nonhydrostatic dispersion

The parameterization currently assumes hydrostatic behavior for all mountain waves. However, nonhydrostatic effects may become important for the narrowest set of ridges currently used (width ~ 25 km) in regions with high wind. When hydrostatic balance is no longer satisfied, mountain wave energy begins to disperse downstream of the generating obstacle as the wave propagates vertically. This causes wave amplitudes to grow more slowly with height than they would for a hydrostatic wave. However, nonhydrostatic wave perturbations also expand progressively farther downstream of the forcing obstacle with altitude. Thus, although nonhydrostatic waves may be weakened, their effects may be felt significantly farther downstream of the forcing obstacle than those of hydrostatic waves. In some cases high winds can completely trap or “turn” vertically propagating waves at some level, creating the familiar lee-wave trains often observed downwind of narrow ridges. These trains of waves are primarily tropospheric phenomena. However, they usually signify that some fraction of the mountain wave spectrum from an obstacle cannot propagate past strong tropospheric winds and thus will not contribute to strato-
spheric wave motion. Simple corrections for nonhydrostatic effects can be derived theoretically and are easily implemented in the a ridge-based parameterization. However, this may require better estimates of actual ridge cross sections and ridge widths than those currently included in the global ridge list.

5. Conclusions

Successful, quantitative forecasts of individual mountain wave events were obtained from a global, ridge-based mountain wave parameterization using relatively coarse meteorological data for background flow conditions. Forecast wave amplitudes for three large wave events were within a factor of 2 of that determined from temperature data collected during ER-2 flights through the waves. The amplitude of small-scale fluctuations in vertical wind observed during passage through the waves was correlated with forecasts of wave momentum flux deposition—wave breaking or wave saturation (Lindzen 1981). That is, turbulence was measured by on-board instruments (and reported by pilots) where the parameterization predicted large, breaking mountain waves. These results suggest that accurate global forecasts of wave-related turbulence can be made as off-line diagnostics using output from existing meteorological forecast models.

The turbulence forecasting algorithm introduced here appears promising compared to techniques based on the local, large-scale Richardson number:

\[ Ri = \frac{N^2}{U^2 + V^2}, \]

where \( N, U, \) and \( V \) represent stratification frequency, zonal wind, and meridional wind, respectively, derived from global forecast models. The strongest zones of turbulence encountered during the mission (14 October 1991 and 18 March 1992) occurred in regions of the atmosphere where \( Ri \) was well above 50. In addition, a significant number of smooth flight miles were logged in zones with relatively low, large-scale Richardson numbers (<30). Finally, forecasts of stratospheric turbulence based on \( Ri \) will necessarily be of coarse spatial resolution, since they depend on flow features with scales of approximately 1000 km. As discussed in section 4a, the large-scale Richardson number may have a role in determining the intensity of turbulence produced during mountain wave breakdown. However, it appears from the limited sample examined here that forecasts of turbulence based on mountain wave characteristics would perform better than \( Ri \)-based forecasts in turbulence/no-turbulence tests for a given flight leg.

All of the turbulent mountain wave encounters examined here took place in the lower stratosphere. However, techniques for predicting wave-induced turbulence similar to those described here should work equally well in the lower atmosphere. A complication in forecasting tropospheric turbulence is that layers of strong shear and low \( Ri \) are more prevalent than in the relatively stable stratosphere. Therefore, unforced, nonorographic turbulence may represent a larger fraction of the turbulence encountered during tropospheric flight. Such turbulence cannot be predicted using the techniques discussed here. Nevertheless, turbulence over orography is frequently encountered in the troposphere as well. Over certain locations, such as the Front Range of Colorado or the eastern slope of the Sierra Nevada in California, mountain wave turbulence poses a well-known and serious threat to aviation. Serious mishaps due to clear-air turbulence, including structural damage to aircraft and loss of aircraft, have been reported in these locations. So, forecasts of mountain wave–related turbulence would clearly be of use to pilots of low- and medium-altitude aircraft.

REFERENCES


